The importance of considering optimal government policy when social norms matter for the private provision of public goods*

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Abstract

In the social norms literature concerned with the private provision of public goods there seems to be an implicit belief that it is best to have all agents adhere to the 'good' social norm. We challenge this view and study optimal government policy in a reference model (Rege, 2004) of public good provision and social approval in a dynamic setting. We show that even if complete adherence to the social norm maximizes social welfare it is by no means necessarily optimal to push society towards it. We stress the different roles of the social externality and the public good problem. We discuss the problem with the standard crowding in and out argument and analyze the relationship with Pigouvian taxes. We discuss the role of the cost of public funds and show how it can create path dependency, multiplicity of both optimal equilibria and optimal paths, and discuss the role of parameter instability.

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1 Introduction

The behavior of individuals is often determined by social norms,[1] which, among others,[2] may help to overcome the underprovision of public goods. These norms can spread or vanish, and often exhibit reinforcing mechanisms where the more people adhere to a norm the more likely is its adoption for an individual. Hence policies that aim to correct for free-riding and externalities should take into account social norms and their dynamic implications. While intuition suggests that it is optimal to push society toward full adherence in case of a 'good' social norm, implying everyone becomes a contributor, this result has not been formally established. The objective of the present article is to understand what is the optimal policy in presence of a public good and a dynamically evolving social norm.

We follow much of the literature and consider that a 'social norm' is a rule of behavior that is enforced through social approval and disapproval (Elster, 1989). As a result of social interactions, social norms can spread through society, and whether or not someone adheres to a norm is the result of personal objectives and social ties. The crucial difference between social norms and legal rules or moral norms is thus the social dimension which may give rise to a potentially self-enforcing dynamic of the norm. Such mechanism can yield multiplicity of equilibria (Young, 2006, 2014) which translates into path dependency and historical lock-in. Concerning public goods, Rege (2004) stresses that a subsidy can

[1]Social norms concern a wide range of behaviors (from tipping to vengeance) and are not always welfare enhancing (e.g. dueling, foot binding). To contribute to a public good is a special case of a 'pro-social' social norm. Social norms have been shown to play a role for littering (Cialdini et al., 1990; Torgler et al., 2009), energy consumption (Allcott, 2011), recycling (Hage et al., 2009; Viscusi et al., 2011), smoking (Nyborg and Rege, 2003b), fuel economy (Yeomans and Herberich, 2014), or tax evasion (Frey and Torgler, 2007; Luttmer and Singhal, 2014). Farrow et al. (2017) provide an overview of the theoretical approaches and the empirical evidence related to pro-environmental behaviors. The dynamic interaction between norms and behaviors has been recently empirically investigated by Huber et al. (2018) for recycling.

[2]Other approaches have been considered, such as altruism (see Warr, 1983; Bergstrom et al., 1986; Sugden, 1982; Andreoni, 1990), moral constraints (Sugden, 1984) or descriptive norms and legal norms or rules (Bicchieri, 2006).

help unlock society from a zero-contributor situation and push it toward a full contribution equilibrium. Nyborg et al. (2006) provide a model of green consumption with social approval that shares similar features. The existence of multiple equilibria associated with social approval has also been studied by Lin and Yang (2006), who argue that only sizable subsidies may induce significant shifts in the equilibria.[3]

While these articles all discuss that both the individuals' decisions and the dynamics of the social norm may usefully be directed through government policy, none of these articles study a fully dynamic, optimal policy intervention. Rege (2004), for example, advocates for temporary policies that help society above a tipping point after which the norm sufficiently penetrates through society such that it develops its own positive dynamics. This view implicitly supposes that there is a cost attached to this policy, as why would one want to stop an otherwise beneficial policy. Thus we take the costly aspect of the policy intervention more serious and in so doing we study whether or not the full contribution equilibrium really is the optimal equilibrium. Based on the model in Rege (2004) we study the optimal government policy intervention where the government maximizes a forward-looking social welfare function taking into account the preferences of the individuals and the dynamics of the social norm. We use Rege's model as, intuitively, it is easy to see why the full contribution equilibrium should be the first best equilibrium. Secondly, it has a simple dynamic structure with a tipping point. This tipping point is the crucial feature in the model without optimal policy and is still important if one considers a simple subsidy, but in the model with optimal policy the tipping point does not play a role any longer.

We follow the literature on the theory of regulation (Laffont and Tirole, 1986, 1993)

[3]The models of Rege (2004); Nyborg et al. (2006); Lin and Yang (2006) does not explain why there is a social norm to contribute but model its dynamics. The approaches of Bicchieri (2006) and Young (2014), or Gordon (1989) and Bethencourt and Kunze (2019) on tax evasion, are similar. Others, within the immense literature on the evolutionary roots of human altruism and pro-social behavior, have tried to model the emergence of social norms as evolutionary stable, fitness enhancing strategies (e.g. Bicchieri et al., 2004; Bowles and Gintis, 2009). Other dynamic approaches include, for example, status seeking Dioikitopoulos et al. (2018).

by assuming a linear cost of public fund due to distortionary taxation (Ramsey, 1927). In addition to the positive externality associated to the public good, 'social' externalities, related to social approval and disapproval, should be taken into account, and the dynamic of the diffusion of the norm plays a crucial role. Our results show that with a linear cost of public funds, full adherence is not optimal. The diffusion slows when the norm spreads, and it is not worth further subsidizing to convert the few last non-contributors. We derive a Ramsey-Pigou formula for the optimal steady state subsidy that encompasses social externalities and a term that could be interpreted as a dynamic price-elasticity.

However, we send a note of caution by giving some first insights into how different assumption about costs of the government policy lead to strikingly different results regarding the optimal level of the social norm and private contributions. With quadratic costs of the public intervention, for instance related to monitoring and enforcement (Kaplow, 1990; De Cara et al., 2018) or administrative costs (Polinsky and Shavell, 1982), it can be optimal to push society toward full adherence with a progressively decreasing public effort. However, there is a phenomenon of path dependency and whether it is worth pushing the norm or letting it vanish depends on the initial distribution of the norm in society.

Several articles have included social dimensions into a static analysis of public good provision In one of the first approaches to model the role of a social norm, Holländer (1990) considers that people care about their relative contribution to a public good and finds that, while social norms induce individuals to contribute to a public good, their chosen allocation would only be second-best. Brekke et al. (2003) assume that people compare their contribution to an ideal one, where the ideal contribution is determined by the social norm. Again, individuals will not choose the first-best allocation as effort to reach this ideal contribution is costly. Bruvoll and Nyborg (2004) analyze the influence of a change of the norm on behavior and welfare, and stress the psychological costs associated to strengthening the norm. Rege (2004) assumes that individuals decide whether or not they want to contribute to a public good given that there is a social norm that arises from interactions with other agents, where contributors approve other contributors and disapprove of non-contributors. This may lead to several potential Nash equilibria, and which one will occur depends on the prevalence of the social norm in society. Bénabou and Tirole (2006, 2011) model the signaling effect of contributing to a public good, altruism being socially rewarded and signaled through high contributions. What these models have in common is that they show that social norms are sometimes helpful to increase contributions to public goods, but that these contributions do not tend to be first best. Consequently, there is room for public policy in order to deal with the public good externality and the social norm.

Researchers have emphasized whether or not a public policy would, via its effect on social norms, crowd in or crowd out private provisions (e.g. Nyborg, 2003; Nyborg and Rege, 2003a; Bowles and Polania-Reyes, 2012). Bowles (2016) discusses in depth the interaction between financial incentives (rewards and sanctions) and pro-social behavior. These articles tend to work in a static framework, thus not taking the externality introduced by the dynamics of the social norm into account. Bowles and Hwang (2008, 2014) consider optimal policy with a public good and social preferences when subsidies have a direct impact on social values (crowding in or out). In their setting, social preferences do not depend on the interaction of individuals. Ulph and Ulph (2018) study static Pigouvian taxes when consumers value conformity in a model of social norms and a public good. [4]

The article is set up as follows. In the next section 2 we start by introducing Rege (2004)'s original model. We then extend her model in section 3 by introducing endogenous government policy. In section 4 we look into further aspects such as a comparison to the Pigouvian tax and in section 5 we study a different public funding cost structure. We furthermore discuss the problems of path dependency and multiplicity of equilibria for optimal policy in section 5. Section 6 concludes.

[4] Similarly, in the related literature on tax evasion and social norms, launched by the seminal article of Gordon (1989), the articles that consider the dynamic of the norm (Dell'Anno, 2009; Bethencourt and Kunze, 2019; Besley et al., 2019) do not analyze optimal government policy. (Bethencourt and Kunze, 2019)

2 The background model

In this section we present the main ideas behind Rege (2004)'s model of a social norm that influences the incentives for the private provision of a public good, discuss the results on the exogenous government policy, and then study the implication of endogenizing this policy.

We fully follow the notation in Rege (2004) for simplicity. Individuals decide whether or not to contribute to a public good. The social norm arises from interactions with other agents, where contributors approve other contributors and disapprove of non-contributors, with the intensity of approval depending on the prevalence of the behavior in society. Adherence to (or rejection of) the norm is not a fully rational decision but the, partly unconscious, result of repeated interactions with contributors or non-contributors.[5] The diffusion of the social norm follows a replicator dynamic. Social approval and disapproval then explain the existence of several steady states and of a tipping point. Rege (2004) then argues that there is a role for public policy which ought to shift society to the full contribution equilibrium.

In Rege (2004) there exists a continuum of agents ($i \in [0, 1]$) who decide to either contribute ($g_i = 1$) or not contribute ($g_i = 0$) to a public good. The share of contributors in society is denoted by $x \in [0, 1]$, and $w(x)$ is an individual's benefit of the public good. This benefit is non-negative, $w(x) \geq 0$, and strictly increasing in the share of contributors, $w'(x) > 0$. Agents have income I that they may use for consumption (at price 1) or for the public good, which comes at price $p > 0$, and they are also affected by a social norm $q_i(x)$. The utility function is assumed to be linear and of the form $U_i = I + w(x) - pg_i + q_i(x)$.

The social norm arises from the interaction of agents. A non-contributor feels disapproval while a contributor feels approval if he is observed by a contributor. In contrast, it is assumed that a person feels neither approval nor disapproval from non-contributors. The magnitude of the feeling of approval or disapproval depends on the frequency of the

[5]That people are unconsciously influenced by social norms has been documented by Nolan et al. (2008) for energy conservation decisions, and more recently by Barth et al. (2016) for electric vehicles adoption.

behavior in society, which is given by $\lambda \equiv w(1) - p > 0$. As defined in Rege (2004), λ denotes "the differences in individual utility, in terms of private consumption, between a society in which everybody contributes and a society in which nobody contributes." Hence, λ is proportional to society's net benefit of the social norm. This net benefit is assumed to be strictly positive, in the sense that if everyone contributes to the social norm then the benefit that everyone obtains exceeds the private cost of contribution. The larger this net benefit the faster will the norm thus evolve through society as individuals will be more inclined to accept it.

The social norm, furthermore, depends on how many agents someone meets from one's own type, the probability of which is $k \in (0, 1/2)$.[6] This yields a social approval of $q_i(x) = \lambda(1-x)(k+(1-k)x)$ for contributors, while non-contributors obtain a disapproval of $q_i(x) = -\lambda x^2(1-k)$.[7] In the static version, individuals play a coordination game in which they choose whether to be a contributor by maximizing their utility, the difference in utility being

$$\Delta U(x) = U^1(x) - U^2(x) = \lambda(k + (1-2k)x) - p. \tag{1}$$

Because of social approval this difference is increasing in the share of contributors. We define $\bar{x}$ to be the share of contributors that makes individuals indifferent between contributing and not contributing, and it is given by

$$\bar{x} \equiv \frac{p - \lambda k}{\lambda(1-2k)}. \tag{2}$$

If the share of contributors is larger than $\bar{x}$, then everyone prefers to be a contributor, while no one prefers to contribute otherwise. As stated in Proposition 1 in Rege (2004), if $\bar{x} \in (0, 1)$ there are three Nash equilibriums of the static game: $x = 0$, $x = 1$ and $x = \bar{x}$. This is the case if the following assumption is fulfilled:

[6] As already pointed out in Rege (2004), for $k \geq 1/2$ the game is no longer a coordination game. We avoid this as the role of the social norm is only prominent in a coordination game.

[7] The components are for a contributor (resp. non contributor): $(1-x)$ (resp. $-x$) is the comparison between the observed behavior and the average; $(k+(1-k)x)$ (resp. $(1-k)x$) the number of contributors met. See Rege (2004) for further details.

Assumption 1 *We assume $p > \lambda k$ and $p < \lambda(1-k)$.*

Rege (2004) took this model a step further and, based on Weibull (1997), Börgers and Sarin (1997) and Taylor and Jonker (1978), allowed the social norm to evolve dynamically and endogenously. Clearly, if agents were fully aware of their preferences and could immediately adopt the social norm, then the social norm would instantly spread through society. This is unlikely to be realistic and it seems more appropriate to assume that agents are not well-informed about their preferences regarding the norm and can only slowly, through repeated interactions, change their behavior. This dynamic evolution can be captured through the so-called replicator dynamics and, denoting time by t and the change in the share of contributors over time by $\dot{x}_t$, is given by the following equation

$$\dot{x}_t = x_t(1-x_t)\Delta U(x_t). \tag{3}$$

Based on these evolutionary dynamics Rege (2004) shows that the three potential equilibria identified above are still possible, but only two are stable ($x_t = 0$ and $x_t = 1$) while $x_t = \bar{x}$ is unstable. This is depicted in Figure 1. The horizontal axis depicts x_t while the vertical axis gives the speed and direction of change of x_t. The thick black line denotes the solution to equation (3). For $x_t < \bar{x}$ society converges over time to the non-contributor equilibrium, while for $x_t > \bar{x}$ society converges to the equilibrium where everyone contributes. In other words, if there are too few contributors in society then non-contributors have not enough incentives to become contributors. Similarly, contributors are not sufficiently approved if they meet too few other contributors, so that it may simply not be worthwhile for them to contribute any longer. As a result, society converges to a norm where nobody contributes. In contrast, if at time $t = 0$ there are sufficient contributors in society then social approval and disapproval motivates non-contributors to become contributors.

Rege (2004) then investigates if the government, by introducing price subsidies s that are paid for via income taxes xs, can instigate a change in society that may induce convergence to the equilibrium with everyone contributing. We thus rewrite the utility of contributors as

$$U^1(x,s) = I - xs + w(x) - p + s + \lambda(k + (1-k)x)(1-x), \tag{4}$$

where $p - s$ is the original price p with the subsidy s, and xs is the income tax, while the utility of non-contributors becomes

$$U^2(x,s) = I - xs + w(x) - \lambda(1-k)x^2. \tag{5}$$

This augments the utility difference, which now becomes

$$\Delta U(x,s) = s + \lambda(k + (1-2k)x) - p. \tag{6}$$

Thus, if the subsidy is large enough ($s > p - \lambda k$) then this utility difference will be positive and society can converge to the high equilibrium. This case is depicted by the dotted line in Figure 1. In this case the previously (instable) equilibrium point $\bar{x}$ is no longer a tipping point. Furthermore, once the subsidy has been put in place for long enough such that the share of the contributors exceeds the tipping threshold ($\bar{x}$), then even without this subsidy society would continue to converge to the equilibrium where everyone contributes to the public good. Thus, a subsidy will crowd-in voluntary contributions.

While it is good to know that government policy can induce changes in equilibria, it is also important to know whether and when this is actually optimal. We now turn to our contribution.

3 The implication of endogenous policy

We now go a step further and, based on the previous model, derive an intertemporal social welfare function that a policy maker would use in order to assess an optimal government policy. We take the simplest possible setting as this already yields the insights that we are after. We assume that agents continue to act myopically when adhering to the norm, and the evolution of the social norm follows the replicator dynamics. However, we assume that there exists a government that can introduce a policy. The policy maker is forward

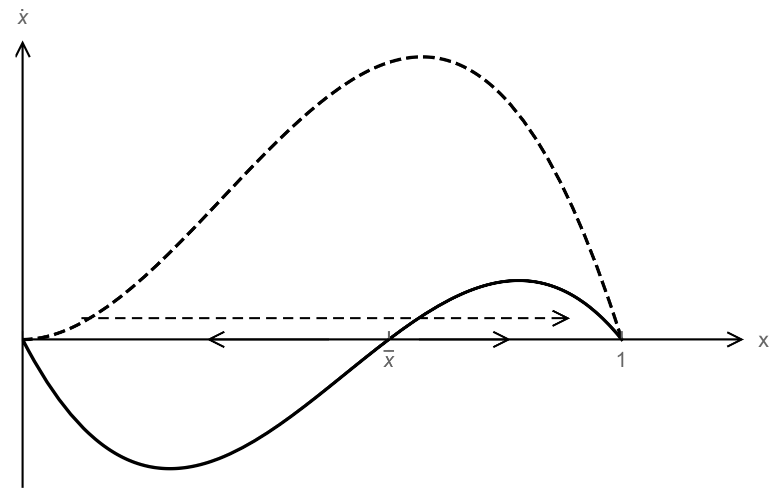


Figure 1: Influence of a fixed subsidy $s > p - \lambda k$ on the dynamic of the share of contributors. Source: Adapted from Rege (2004).

looking, has perfect foresight, and maximizes the infinite stream of the agents' utilities by appropriately setting incentives. The policy maker can give a subsidy s_t to contributors. We allow this subsidy to be negative, which gives the policy maker complete freedom over the direction in which he wants to push the production of the public good as well as the social norm. In order to balance the budget, the policy maker raises taxes via either lump sum or income taxation.

Assumption 2 *We assume the existence of exogenous bounds on s_t in the form of $\underline{s} \leq s_t \leq \bar{s}$, with $\underline{s} < 0$. We shall, for simplicity, assume that $\underline{s} \leq p - \lambda(1-k)$ and $\bar{s} \geq p - \lambda k$.*

These bounds are a natural restriction for public policy. We assume them to be large enough so that the policy maker is assured full flexibility over the influence of the policy.[8]

[8] In case one believes the assumption of these bounds to be too restrictive, then it can readily be argued that public debt will help to alleviate the exogenous bounds on the policy. Let us, for example, take the case where $\bar{s} < p - \lambda k$. In this situation the subsidy is not large enough to induce a positive utility difference even for low levels of the social norm. As a result, the policy maker cannot induce a change in the social norm and public policy would not have any effect. However, imagine now that the policy

Then based on the model introduced in the previous section we assume that the policy maker fully takes the preferences of the individuals as given and thus maximizes a Bergson-Samuelson welfare function (Samuelson, 1977), where the *instantaneous* gross welfare is given by the sum of agents' utilities, $V = xU^1(x,s) + (1-x)U^2(x,s)$ which only depends on x and after substitution this yields

$$V(x) = I + w(x) - px + \lambda x(1-x)k. \tag{7}$$

One issue concerns the question whether or not the government should include the social norm in the individuals' utilities that make up the social welfare function. The traditional Bergson-Samuelson welfare function (Samuelson, 1977) takes the individuals' preferences as given, revealed by their choices. Some argue that observed choices do not systematically maximize the true utility of agents, and introduce the distinction between decision utility and experience utility. The latter should then be the objective of the social planner(e.g. Kahneman et al., 1997; Thaler and Sunstein, 2003; Loewenstein and Ubel, 2008) . We consider that there is no such tension in our case. Individuals experience approval or disapproval, these are associated with real positive and negative emotions that should be part of their experience utility. However, in some cases social effects could have an unconscious influence on choices (similar to a framing effect) and may therefore not be included in the welfare function. We shall come back to this issue when discussing the influence of the social norm on the optimal share of contributors in Section 4.

To gain some intuition on the influence of the norm within this welfare function setting, we take the derivative with respect to x, slightly rewrite it by making use of equation (6), and obtain

$$V'(x) = w'(x) + \Delta U(x,s) - s + \lambda \left\{ x\Big[(1-k)(1-x) - (k+(1-k)x)\Big] - (1-x)2(1-k)x \right\}. \tag{8}$$

The term in curly brackets is the marginal external social impact of an increase in the share of contributors. It is the sum of the effect on contributors and on non-contributors,

maker can raise public debt and thereby endogenize these bounds. If this debt comes at a sufficiently low price (interest rate), then the policy maker would clearly be inclined to raise debt in order to be able to influence the norm.

which derive from the increased probability of meeting a contributor and the influence on approval intensity. For non-contributors, the diffusion of the contributory behavior has a cost only: they meet more contributors and feel more disapproval. For contributors the effect is ambiguous as they benefit from meeting more contributors but their feeling of approval is reduced for each encounter. After simplification, the term in curly brackets reduces to $-\lambda x$, implying that, at the equilibrium when we expect $\Delta U(x,s) = 0$, social approval and disapproval represent an external cost.

In each period the policy maker balances the budget but whenever he raises taxes to pay for the price subsidy then he incurs a deadweight loss. We represent this cost of public funds by $\gamma > 0$, i.e. raising \$1 of public money costs society $\$(1+\gamma)$ because of distortionary taxation (Pigou, 1947; Ramsey, 1927; Stiglitz and Dasgupta, 1971; Atkinson and Stern, 1974). Conversely, if the government taxes the public good ($s < 0$) then this reduces the taxpayers' burden by $sx(\gamma+1)$.[9] This assumption is standard in the theory of regulation (Laffont and Tirole, 1986, 1993; Laffont, 2005).[10]

Hence we can now write the policy maker's objective function, which is given by

$$W = \int_{t=0}^{+\infty} e^{-\phi t} \big[V(x_t) - \gamma x_t s_t \big] dt, \tag{9}$$

where $\phi > 0$ is the discount rate. Based on this setup there is only one state equation

$$\dot{x} = x(1-x)\Delta U(x,s). \tag{10}$$

The policy maker then maximizes equation (9) with respect to s_t, subject to the constraint (10) and with bounds on subsidies in the form of $\underline{s} \leq s_t \leq \bar{s}$.

[9]It is also possible to introduce a tax on non-contributors, if such a tax t is available, the difference in utility becomes $\Delta U(x, s+t)$ and the total budget$xs - (1-x)t = x(s+t) - t$, it is *as if* every agent was taxed t and contributors receive a subsidy $s+t$ which becomes the relevant variable to be set by the regulator.

[10]We should add that this assumption is debated in the optimal taxation literature. For example, Jacobs (2018) establishes that the cost of public funds should be zero at the optimal tax, if lump sum taxation is available (see also Jacobs and De Mooij, 2015, on Pigouvian taxation), while it can be positive for sub-optimal tax systems in second best frameworks. Conclusively, our assumption should be understood for the more empirically relevant case of a sub-optimal tax system.

Using equation (6) and substituting s into the expression (9) of the objective function yields

$$W = \int_{t=0}^{\infty} e^{-\phi t} \left\{ V(x) - \gamma x_t \Big[p - \lambda(k + (1-2k)x_t) \Big] - \gamma \frac{\dot{x}}{1-x} \right\} dt \tag{11}$$

Due to the linearity of the control s_t we know that the solution to this is a *Most Rapid Approach Path* (see Kamien and Schwartz, 2012, Chapter 16). Maximizing the objective function we obtain the Euler equation, and denoting the optimal solution by x^*, we then get

$$V'(x^*) - \gamma \Big[p - \lambda k - 2\lambda(1-2k)x^* \Big] - \frac{\gamma\phi}{1-x^*} = 0. \tag{12}$$

This optimal level of contributors depends on three terms. The first is the effect on per period welfare $V(x)$ of an increase in the number of contributors. This term can be further decomposed as $V'(x) = w'(x) - p + \lambda(1-2x)k$, the sum of the marginal benefits from the public good, the cost of contributing and the marginal social approval effect. Note that this last social effect can be either positive or negative depending on how many people are already contributing to the public good. The second term is the effect on the per period cost of the total subsidy xs. The subsidy necessary to sustain a given share of contributors is decreasing in the share of contributors, an effect which is due to the social approval. The third term is related to the impatience of the social planner, it is the discounted cost of the subsidy. The more impatient the social planner the lower the benefits from increasing the number of contributors. This marginal cost grows to infinity as x becomes close to 1, for then the norm spreads slowly in the society (there are few non-contributors to be converted) and it becomes very costly to further incentivize agents to take up the norm. This time effect crucially hinges on the existence of the deadweight loss that the policy maker bears each instance.

Assumption 3 *We assume that* $w'(0) > \gamma\phi + (1+\gamma)(p - \lambda k)$ *and* $w'''(x) \leq 0$.

This insures that there is a unique interior solution to x^*. While this assumption is not necessary and only sufficient, it simplifies the subsequent analysis and allows us to focus more clearly on the essential results. In section 5 we discuss the issue of multiplicity more deeply.

Proposition 1 *The optimal solution to the maximization problem* (11) *is a* Most Rapid Approach Path. *Given Assumptions 1, 2 and 3, the optimal policy consists of:*

- *If $x_t < x^*$ then $s_t = \bar{s}$, and $\dot{x} > 0$;*
- *If $x_t > x^*$ then $s_t = \underline{s}$, and $\dot{x} < 0$;*
- *And once $x_t = x^*$ the steady state solution is*

$$s^* = p - \lambda(k + (1 - 2k)x^*) \tag{13}$$

in which x^* solves (12)*.*

Proof 1 Due to the linearity of the control s_t the solution to the optimal control problem is a Most Rapid Approach Path (see Kamien and Schwartz, 2012, Chapter 16). The Euler equation then is given by (12). The properties of this Euler equation are as follows. Define $SL(x) \equiv w'(x) - (1 + \gamma)(p - \lambda k) - 2\lambda x(k - \gamma(1 - 2k))$, and $SR(x) \equiv \frac{\gamma\phi}{1-x}$. Then we obtain $SR(0) = \gamma\phi > 0$, $SR(1) = \infty$, $SR'(x) = \frac{\gamma\phi}{(1-x)^2} > 0$ and $SR''(x) = 2\frac{\gamma\phi}{(1-x)^3} > 0$. Furthermore, $SL(0) = w'(0) - (1 + \gamma)(p - \lambda k)$, $SL(1) = w'(1) - (1 + \gamma)(p - \lambda k) - 2\lambda(k - \gamma(1 - 2k))$, $SL'(x) = w''(x) - 2\lambda(k - \gamma(1 - 2k))$ and $SL''(x) = w'''(x)$. Thus by Assumption 3 $SL(0) > SR(0)$ and $SL''(x) \leq 0$, there is at least one equilibrium since $SL(1) < SR(1)$, and it is unique since $SR'(x) > 0 > SL''(x)$. Then, for $x_t < x^*$ we have $s_t = \bar{s}$, while for $x_t > x^*$ we obtain $s_t = \underline{s}$. For $x_t = x^*$ we have the steady state solution $s^* = p - \lambda(k + (1 - 2k)x^*)$, where x^* solves (12). ■

At the steady state the optimal subsidy can be positive or negative. If it is positive, the cost of further increasing the subsidy should be equalized with the discounted value of the benefits from increasing the share of contributors. If it is negative, the public good is taxed and the benefits from this tax should be compared with the losses from decreasing the pool of contributors. We can directly see that $s^* < 0$ *if and only if* $x^* > \bar{x}$. More precisely, the policy maker applies a tax if $x_t > x^*$, or a subsidy if $x_t < x^*$. This stands in stark contrast to the general belief that government policy should enforce the full contribution equilibrium and where thus a tax (a negative subsidy) was never even considered. However, in our case the government understands that a tax on the price would help reduce the deadweight loss from other policies elsewhere and this thus provides incentives for the government to not induce the $x = 1$ equilibrium.[11] There are obviously further ways in which the costs of the government policy can be introduced in a model, and in section 5 we look more closely into this. We shall show that this result may persist but then depends on parameter configurations.

Two results related to the dynamics are worth stressing: First, the government policy enforces the optimal level of contributors, x^*, and any deviation would trigger an adjustment of the subsidy to ensure that society comes back to this optimal level. The original social norm tipping point $\bar{x}$ is no longer tipping the system since now the policy maker directs the social norm. Indeed, x^* would be the tipping point associated to s^*, if s^* were fixed once and for all. This is different from the result in Rege (2004) where the policy was neither assumed to be costly nor distortionary. At the optimal policy the system is no longer tipping.

Second, it is never optimal to push society toward $x = 1$ because of the cost of public fund and the diffusion dynamics. If x is close to 1 then there are few non-contributors left, and it is too expensive to subsidize contribution to convert them into contributors. Conversely, to tax contributors has a small effect on the dynamic of the norm and is then justified by the cost of public funds.

We now show that, despite the fact that the full contribution equilibrium maximizes social welfare, the presence of the costly public policy may (this depends on the shape of the deadweight loss as we show later) imply that this is finally not the policy maker's preferred equilibrium. For this we define the first best share of contributors as the share x^{FB} that maximizes $V(x)$ defined by equation (7).

Assumption 4 *$w'(1) - p > \lambda k$.*

[11] It must be emphasized that this result arises from a reduction in the deadweight loss that is outside of this model, but without loss to this main result we could easily extend the model to encompass this in a more general equilibrium setting.

Hence we assume that we are in a world where full adherence to the norm seems socially desirable, and, by itself, the social norm can potentially move society toward this situation if $x(0) > \bar{x}$.[12] In such a case, the full contribution equilibrium is desirable.

However, this is not necessarily true once we consider costly regulatory intervention. With costly policy, the optimal share of contributors becomes contingent on the regulatory tools used. More specifically, even if Assumption 4 is true then despite this our result above shows that the optimal equilibrium is below the full contribution one, $x^* < x^{FB} = 1$, and thus our regulator never pushes society toward the $x = 1$ equilibrium.[13]

We thus conclude that, despite the assumption that the full contribution equilibrium is socially optimal without government intervention, the existence of this policy intervention changes this result. The full contribution equilibrium, for which authors in the literature on the private contribution of public goods suggest government intervention (Rege (2004); Nyborg et al. (2006); Lin and Yang (2006)), may not be optimal any longer once one considers (costly) government intervention.

4 Relation with Pigouvian tax

In general, research that deals with public intervention and social norms discusses whether or not the public policy has a crowding in or crowding out effect. One of the reasons for the focus on crowding in and out is that it is acknowledged that actions undertaken due to social norms have an intrinsically superior value than the same actions induced by public interventions. Furthermore, the argument stands that under crowding out of the social norm a public policy may only have very limited impact in general.

We believe this focus is too limited because it ignores the nature of public interventions,

[12] Note that with $\lambda = w(1) - p$, as assumed by Rege (2004), and $k < 1/2$, the above assumption is satisfied if $w(x)$ is linear with respect to x (which is not the case in Rege (2004)).

[13] If one assumes the absence of costly public funds, the regulator sets a high subsidy to move society toward $x = 1$ as fast as possible whether there is a social norm or not, convergence is accelerated by the social norm which is welfare improving.

namely to correct market failures in addition to inducing socially optimal outcomes. Based on this line of thought we here make the bridge from the crowding in and out literature to the Pigouvian tax literature. The textbook way to correct for externalities is a Pigouvian tax. In the model that we study a policy maker faces two externalities, the externality of the public good and the one of the social norm.[14] The positive externality associated with contribution to the public good justifies a subsidy. At the same time the social norm may help to overcome the free-riding problem, but it also introduces a social externalities related to social approval and disapproval. Thus it needs to be optimally managed as well.

Some readers may argue now that we endow the policy maker with only one instrument to deal with both externalities. However, this is precisely where Rege (2004)'s model comes in handy. Both the externality of the social norm and the public good problem are defined by one variable only, namely x. In other words, both externalities only depend on the single variable x. Hence one policy tool is sufficient in this case.

We now relate our previous results to Pigouvian taxes. We can only do this comparison at equilibrium where $x_t = x^*$, and thus constant, simply because we know that during transition the MRAP implies that s_t will be at either its lower or upper bound. In order to clarify the relationship with a Pigouvian tax we combine equation (12) and (13), assuming there exists an interior solution to equation (12), to obtain

$$s^* = \frac{w'(x^*) - \lambda x^*}{1+\gamma} - \frac{\gamma}{1+\gamma}\left(\frac{\phi}{1-x^*} - \lambda(1-2k)x^*\right). \tag{14}$$

The formula we obtain here is very much akin to the Ramsey formula of optimal taxation.[15] The term $(w'(x) - \lambda x)/(1+\gamma)$ is the marginal benefit from the public good in public monetary units, its difference with the optimal subsidy being the implicit tax

[14]While our policy maker wants to deal with those two externalities his public policy introduces a third market failure, namely a deadweight loss or gain due to the public funds. We will study the implication of not having this externality, but for a smoother working of the model we require costly public funds, otherwise there would never be an interior solution to the model. Plus, of course, it is a realistic feature of the model.

[15]For $\phi = 0$, Equation (14) can be rewritten in the Lerner-Ramsey form, defining $\epsilon = (p - s^*)/(\lambda(1 - 2k)x^*)$ (the elasticity of the steady state share of contributors with respect to the cost of contributing),

on contributory behavior. Thus, s^* can then be related to the standard Pigouvian tax (or subsidy) at equilibrium. There are three components that play a role: the marginal benefit of the public good, the costs (or benefits) of public funds, and the social norm. We now study the role of the different components.

First, we set λ and γ equal to zero, and denote the optimal solution in this case as $x^*_{\gamma\lambda}$. This case corresponds to one where public funds do not come at a cost and the social norm does not evolve. Equation (14) is then simply $s^*_{\gamma\lambda} = w'(x^*_{\gamma\lambda})$, and the optimal subsidy is the Pigouvian subsidy which is equal to the marginal external benefit, and at equilibrium obviously also equal to the price p.[16] This extreme case corresponds to a world without a social norm and without costly government funds and it prescribes an optimal Pigouvian tax along the lines of the standard public good literature (Samuelson, 1954). Note that, under Assumption 4, thus if $w'(1) > p$, then it is clear that the optimal solution would be to have $s_t = \bar{s}$, $\forall t$. In this case the corner $x = 1$ will be approached over time and the policy maker makes sure that society stays there.

Let us align this point more closely with the public goods literature that considers costly government funds, where we now assume that $\gamma > 0$ but still ignore the evolution of the social norm such that $\lambda = 0$. Hence, the policy maker would want to achieve an optimal level of the public good x_λ corresponding to

$$w'(x_\lambda) = \frac{\gamma\phi}{1 - x_\lambda} + (1+\gamma)p.$$

In this case $w'(x_\lambda) > p$ and hence the solution for x_λ will be lower compared to $x_{\gamma\lambda}$ as now the costs of the public funds make policy intervention at equilibrium more costly.

then

$$\frac{1}{p - s^*}\Big[\frac{w'(x^*) - \lambda x^*}{1+\gamma} - s^*\Big] = -\frac{\gamma}{1+\gamma}\frac{1}{\epsilon}.$$

[16] It is clear that this result is not fully mathematically correct but corresponds more to a limiting case. To be precise, if $\gamma = \lambda = 0$, then welfare W will be maximized at $w'(x) = p$. As there is now no cost to policy, then the optimal policy will be a bang-bang solution, with $s_t = \bar{s}$ for $x_t < (w')^{-1}(p)$, $s_t = s^*_{\gamma\lambda}$ for $x_t = (w')^{-1}(p)$, and $s_t = \underline{s}$ otherwise. This result applies since the government knows that equation 6 still holds, meaning that agents decide according to the utility differences.

Thus the costs of public funds essentially create a wedge between the price of the public good p and the marginal benefit to each agent. In addition, and more importantly, these costs interact with the dynamic diffusion of the norm which explains the presence of the discount rate in the formula above. As is also clear, it is not optimal to push society towards the full contribution equilibrium, which implies that the result $x^* < 1$ is not directly related to the presence of the social norm but the combination of the replicator dynamics and the cost of public funds.

Assume now that the social norm plays a role, $\lambda > 0$, but for clarity that the cost of public funding is zero, $\gamma = 0$, such that equation (14) becomes $s^*_\gamma = w'(x^*_\gamma) - \lambda x^*_\gamma$. From equation (8) we know that $-\lambda x$ is the social external cost associated with the social norm at the intertemporal equilibrium. In this case the optimal subsidy encompasses two Pigouvian terms, the external benefit of the public good, as well as the social external cost. However, social benefits and costs also play another role due to the utility difference. Even though the optimal subsidy is lower with the social norm than without it the optimal share of contributors might well be larger with the social norm.[17]

Whether the presence of the social norm justifies a higher or lower optimal share of contributors depends on several factors, the negative social externality but higher internalized incentive to contribute, and, in addition, the lower regulatory costs when $\gamma > 0$. The optimal share of contributors can still be larger with the social norm than without it. It is illustrated in Figure 2, in which x^* and x_λ are depicted as a function of the cost of public fund. In the Figure, Assumption 4 is satisfied ($x^{FB} = 1$). For a small cost of public funds, the optimal share of contributors is larger without the social norm than with it. The benefits associated with the lower subsidy allowed by the social norm are not sufficient to compensate for the social external costs.[18] The comparison is reversed for large costs of public funds.

[17]The total social marginal benefits of an increase of x is $\lambda(1 - 2x)k$, of which $\lambda[(1 - 2x)k + x]$ are internalized when $\Delta U = 0$, and $-\lambda x$ are external costs.

[18]Note that for $x^* > \bar{x}$ the equilibrium subsidy is positive.

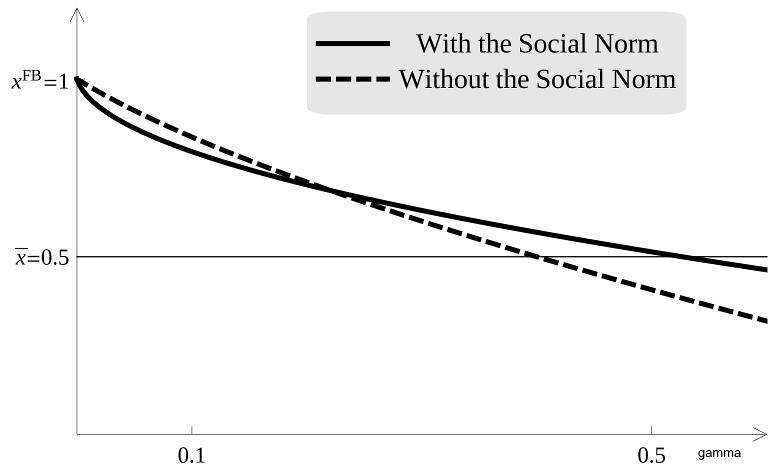


Figure 2: Optimal share of contributors as a function of the costs of public funds with the social norm (thick line) and without it (dashed line). The figure is obtained for $w(x) = (2 - x/2)x$, $p = 0.5$, $k = 0.4$, $\lambda = w(1) - p = 1$, $\phi = 1$

To conclude our discussion on the influence of the social norm let us consider a "technocratic" social planner who does not take into account approval and disapproval feelings in the welfare evaluation but correctly anticipates the influence of the social norm on the diffusion of the behavior. He will thus maximize

$$W = \int_0^{+\infty} e^{-\phi t}\Big[(w(x) - px) - \gamma x s\Big] dt \tag{15}$$

The optimal solution to this is given by a Most Rapid Approach Path toward an optimal steady state level x^{**} that solves

$$w'(x^{**}) - p - \gamma\Big[p - \lambda k - 2\lambda(1-2k)x^{**}\Big] = \frac{\gamma\phi}{1-x} \tag{16}$$

The optimal steady state level x^{**} is larger than x^* (the steady state solution if the planner takes approval and disapproval into account) if and only if $x^* > 1/2$.[19]

[19] x^* solves equation (12) so: $w'(x^*) - p - \gamma[p - \lambda k - 2\lambda(1-2k)x^*] - \gamma\phi/(1-x^*) = \lambda k(2x^* - 1)$, therefore $x^* < x^{**}$ iff $2x^* > 1$.

The objective function (15) can be justified if individuals are unconsciously influenced by the social norm but do not consciously experience any positive or negative feelings. The objective is then to maximize the experience utility and not the decision utility. The social effects $\lambda x(1-x)k$, the third term in the expression (7) of V, are not valued by the technocratic social planner. These social effects are maximized for $x = 1/2$ which explain the threshold obtained. Thus there will obviously be a difference between the optimal solution when a planner relies on experience utility or choice utility. The optimal solution in the case of experience utility will be larger than that for the decision utility (as long as $x^* > 1/2$) as the (technocratic) planner does not take the social approval and disapproval into account.

5 Administrative costs

In the preceding sections, we made the common assumption of a linear cost of public funds. This linearity led to the result that it is never optimal to push society towards the full contribution equilibrium. This assumption is justified in a partial equilibrium setting when the cost mainly comes from deadweight losses associated with the taxes (e.g. Laffont, 2005). In contrast, in a more general equilibrium setting, one can consider that whether the policy maker introduces a tax or a subsidy, there are administrative costs related to enforcement and tax collection which lead to an increasing marginal cost of public funds (Kaplow, 1990; Polinsky and Shavell, 1982; Bowles and Hwang, 2008).

Thus, our objective here is threefold. First, since we want to motivate readers to further investigate the role of optimal policy in the social norm literature, we want to show that assuming linear or non-linear costs of public interventions can lead to substantial differences in the results. For example, we shall show that moving to a non-linear modeling of the cost can make the full contribution social norm level, in contrast to the linear case, an optimal equilibrium. Second, our motivation is to move away from this somewhat partial equilibrium argument that founded our linear deadweight loss model, towards a general equilibrium model. While this may be an argument of semantics mostly, it may be a more appealing setting to the macroeconomic readership. Third, this non-linear case allows us to derive additional results regarding the choice to move between equilibria and the importance of initial conditions.

Let us assume that there is a cost to collect subsidies that is a quadratic function of the subsidy per individual so that the total cost is now given by $x\gamma s^2/2$.[20] The objective of the social planner is then to maximize

$$W_t = \int_{t=0}^{\infty} \Big(V(x_t) - \gamma x_t s_t^2/2 \Big) e^{-\phi t}. \tag{17}$$

The policy maker then maximizes equation (17) subject to s_t and the constraint (10). We delegate the derivations to the Appendix and only present the main results here.

After maximization we can derive a system of differential equations in $\{x, s\}$, which is given by

$$\begin{aligned}
\dot{x} &= x(1-x)\Big(s - p + \lambda(k + (1-2k)x)\Big), & (18)\\
\dot{s} &= -xs\big(s - p + \lambda(k + (1-2k)x)\big) & \\
&\quad + s\big(\phi - (1-2x)\big(s - p + \lambda k\big) - (2-3x)\lambda(1-2k)x\big) & (19)\\
&\quad - \frac{1-x}{\gamma}\Big(w'(x) - p - \lambda(1-2x)k - \gamma s^2/2\Big). &
\end{aligned}$$

This system completely describes the dynamics of x_t and s_t. It gives rise to three potential candidates for steady states.

The first candidate is the $x = 0$ equilibrium. While we know that $x_t = 0$ is one of the potential steady state solutions for $\dot{x}_t = 0$, we also know that s_t at $x_t = 0$ is a variable that the policy maker can choose freely as it does not entail a social cost. We need to figure out whether the dynamic system would make us approach this steady state. In the convergence to this steady state the necessary conditions must be fulfilled. Whether

[20] One suggestion that we came across is to use the form $x(\gamma_0 s + \gamma_1 s^2)$ in order to link the linear and the quadratic case. One problem here is that this functional form only makes sense for positive s. Furthermore, one should expect administrative costs to be null for $s = 0$, and it will be clear that $x^* = 1$ is only possible for $\gamma_0 = 0$, the marginal cost should progressively converge toward zero.

or not convergence to this steady state is optimal will depend on the shape of the phase curves and thus the associated dynamics.

Whether it can be optimal locally, for small initial level of the norm, to go toward $x = 0$ depends notably on $w'(0)$. If the marginal benefit from the public good is large at $x = 0$ it is then never optimal, whatever x_0, to go toward $x = 0$, the policy maker should then sufficiently subsidize the behavior to help overcome the social trap. Conversely, if $w'(0)$ is small, then the policy maker should not prevent the norm from disappearing, but should still set a subsidy in order to slow the process (see Appendix for further analysis). It is illustrated in Figure 4b in which for small initial x_0 the green trajectory should be followed.

The second candidate is the equilibrium of $x = 1$. This is the steady state where the policy maker would push for the highest level of the social norm in society. Substituting the $x_t = 1$ solution into the dynamic system yields the logical optimal solution $s_t = 0$. We know that from the threshold level $\bar{x}$ onwards the social norm is self-enforcing, then it is clear that positive subsidies after x_t crossed this threshold are only necessary in order to push x_t faster towards its steady state. Intuitively, the reason for a positive level of s_t for $x_t > \bar{x}$ is then only that the deadweight loss is small compared to the higher social norm (which would anyway have occurred).

The Jacobian around the $\{x_2, s_2\} = \{1, 0\}$ steady state is given by

$$\mathcal{J}\Big|_{(x_2,s_2)} = \begin{bmatrix} p - (1-k)\lambda & 0 \\ \frac{w'(1)-p-k\lambda}{\gamma} & \phi \end{bmatrix}.$$

As this is a lower triangular matrix we have that the eigenvalues are given by $EV_1 = p - (1-k)\lambda$ and $EV_2 = \phi$. This steady state is saddle path stable if $p < (1-k)\lambda$, which applies given Assumption 1. Conclusively, it is now possible that this steady state is optimal, which stands in stark contrast to the linear deadweight loss case.

The intuition is restored with the quadratic cost specification: it can be optimal (at least locally) to introduce a subsidy to stimulate the norm and then progressively phase out the policy as the norm spreads through society. The explanation for the difference between the quadratic and linear cost is related to the marginal cost to push the norm as the x gets close to 1. As x converges toward 1, the subsidy is progressively reduces to zero and so does the marginal cost per contributor γs, consequently the annualized cost to push the norm $\gamma s/(1-x)$ stays bounded in contrast to the linear cost case.

Hence, we find that not only is it important to acknowledge that there is a wider need to study the implication of optimal policy in the social norms literature, but we also observe that the way we model this public intervention can yield vastly different results given the optimal strategy that a policy maker may want to pursue.

The third candidate is the interior equilibrium characterized by $\Delta U(x, s) = 0$, that is, $s = p - \lambda(k + (1-2k)x)$. Substitute this into equation (20) evaluated at steady state, using $V'(x) = w'(x) - p + \lambda(1-2x)k$ and defining $s(x) = p - \lambda(k + (1-2k)x)$, gives us

$$V'(x) - \gamma\Big[\frac{1}{2}s(x)^2 + xs(x)s'(x)\Big] = \frac{\gamma s(x)\phi}{1-x}. \tag{20}$$

The parallel with equation (12) is then clear, the bracketed term is the derivative of the public cost $\gamma x s(x)^2/2$. The right hand side is the annualized marginal cost of an increase of x,and contrary to the linear cost case, it is proportional to the subsidy s.

This interior steady state equation is rather complicated and allows for a multitude of combinations of interior steady states with a wide variety of dynamics. The interior steady state can be unique and stable or unstable, or there can exist interior multiple steady states which are stable or unstable with or without complex dynamics, or it is also possible that no interior steady state exists at all. Finally, there is the possibility of Skiba points, such that there exists an initial condition $x(0)$ for which it is optimal to converge to either of the various equilibria. We discuss this in the next section.

Path dependency and parameter stability

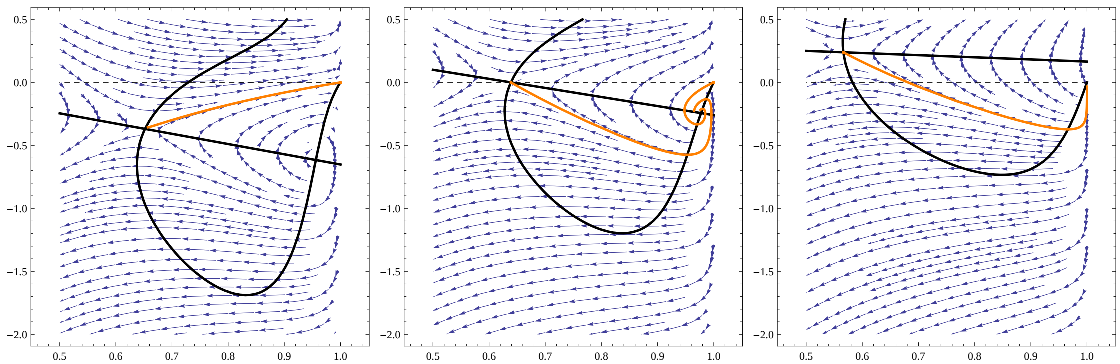
One issue that we have so far avoided is path dependency and parameter stability. It is a well-known result that initial conditions matter already without government intervention. For example, as Rege (2004) has shown, if the initial distribution of the social norm in society is favorable (meaning $x_0 > \bar{x}$), then society will converge to the full contribution equilibrium. Thus, whatever path led society to this initial condition, its subsequent evolution is fully depending on that level.

At the same time, it is clear that society first needs to develop a certain social norm, and these developments need to be done from scratch. In other words, society would be expected to start around the $x = 0$ equilibrium. Thus, for many social norms that have similar qualitative features such as ours, one can expect that, without some further incentives to initially adopt the norm, then no one in society would ever adhere to it. This, obviously holds especially true for the type of norm as developed in Rege (2004), where for a low initial distribution of the norm ($x_0 < \bar{x}$) society would never adopt it without something that provides further impetus of some sort.

In our extension above we have argued that public policy may want to provide such an incentive, and this incentive should be introduced in a socially-optimal way. As we argued, depending on parameters and functional forms, a policy maker would find it optimal to make society adhere fully to the social norm, to have no one adhere to the social norm, or any conceivable intermediate result. While it may be possible for a well-informed policy maker to know what would be the optimal policy, we shall now present a further complication which makes it very difficult to judge as to what is the correct policy. We shall do this with the social welfare function, equation (17), with the squared costs in mind.

As we have argued above, in the case of the squared costs we can easily identify a variety of potential steady states,[21] some of which have properties that give difficulties to policy choices. To be specific, let us look at Figures 3a to 3c. Note that x is only depicted between 0.5 and 1. As we can see, there are at maximum three potential equilibria, one being the corner equilibrium ($x = 1$, $s = 0$), the other being a saddle-path stable interior equilibrium, and between this interior equilibrium and the corner equilibrium there may exist an unstable spiral equilibrium. In the case of Figure 3b, around this unstable spiral equilibrium there is a Skiba point (see e.g. Wagener, 2003). A Skiba point is an initial share of contributors x_0, such that it is optimal to converge to either of both surrounding

[21]Multiple steady states are also possible with the linear cost case, but we have yet to be able to show the existence of bifurcations and other qualitative changes to the dynamics in that case.



(a) Figure 3a (b) Figure 3b (c) Figure 3c

Figure 3: Phase diagrams for the dynamic system derived from (17) with constraint (10). On the x-axis we have x_t, on the y-axis s_t. The full thick lines denote the $\dot{x}_t = 0$ and $\dot{s}_t = 0$ phase curves, the orange lines show the stable manifolds. We assume $w(x) = ax^b$. In Figure 3a we use the parameters $a = 2.8$, $b = 0.3$, $p = 0.7695$, $k = 0.3$, $\gamma = 1$, $\phi = 0.08$. The unstable manifold of the interior steady state connects with the stable manifold of the corner steady state. In Figure 3b we use the parameters $a = 2.8$, $b = 0.3$, $p = 1$, $k = 0.3$, $\gamma = 1$, $\phi = 0.08$. There is a Skiba point around the unstable, spiraling interior steady state. In Figure 3c we use the parameters $a = 2.8$, $b = 0.3$, $p = 1.1$, $k = 0.45$, $\gamma = 1$, $\phi = 0.08$. The unstable manifold of the corner steady state connects with the stable manifold of the interior steady state.

stable equilibria (the saddle-path stable interior equilibrium or the corner). However, for a small change in the initial condition to the left or right, only one of the equilibria is optimal. This is an example of a path dependency which shows that it is vital for a policy maker to precisely know the depth of the social norm throughout society.

Taking this a step further, it also means that there is a historical lock-in, or social trap, even from the perspective of a policy maker searching for an optimal policy. Thus, for a low initial distribution of the social norm the policy maker may not find it optimal to induce a high equilibrium in society as it is simply too costly or time intensive. Instead, in this case the policy maker may view a low distribution of the social norm as socially optimal. Hence, social traps may be the result of this setting.

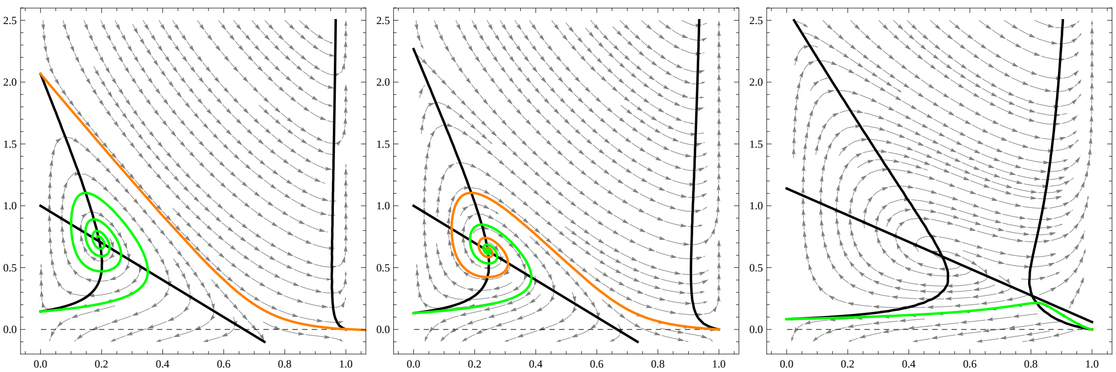
Furthermore, for a small change in parameters the system can undergo a significant qualitative change (a bifurcation), implying that a previously targeted equilibrium may no longer be optimal. Hence, it is vital to have a very precise estimate of the parameters, as any smaller variation in a parameter could mean that the policy maker has to completely reverse his previous policy. For example, moving from Figure 3a to Figure 3b we only changed k by 0.15, but this changed the optimal equilibrium from the corner one to having both the corner and the saddle-path stable interior equilibrium being optimal. Another smaller change in k (by 0.05) implies that the interior state becomes optimal. Conclusively, both a deep knowledge on the structural parameters as well as a sufficient parameter stability seem essential for policy intervention to be successful.

A few comments may help interpret the Figure 3. First, with the specification used ($w'(0) = +\infty$) it is never optimal to converge towards $x = 0$. For small initial values of the norm the policy maker implements a subsidy that is progressively reduced until x reaches the first (small x) equilibrium. Concerning the $x = 1$ equilibrium, $w'(1)$ is smaller than p which means that without costly public fund it would not be optimal to implement $x = 1$.[22] It is why the $\dot{s} = 0$ curve reaches point $(x = 1, s = 0)$ from below which corresponds to a tax.[23] The policy maker actually slows the diffusion of a too strong social norm if the norm is already well developed, he implements a small tax that is progressively reduces (higher branch of the spiral). For intermediate initial values of the norm, he reduces it by setting a large tax (lower branch of the spiral), which is also progressively reduces. For an initially low distribution of the norm it is optimal to subsidize it and progressively reduce the subsidy.

Technically to get a spiral and a Skiba point around the $x = 1$ equilibrium one needs that $w'(1) < (p - \lambda k)$ (so that the $\dot{s} = 0$ curve cross the $\dot{x} = 0$ curve from above) and the policy associated is a tax near the corner. However, it is also possible to get a spiral and complex dynamics with a subsidy along the optimal trajectory as illustrated in Figure 4, for the case of a linear $w(x)$.

[22]With the specification used for the Figure 3: $w'(1) = b \times a = 0.3 \times 2.8 = 0.84$ which is smaller than p.

[23]Locally, at $x = 1$, the derivative with respect to x of the curve $\dot{s} = 0$ is $-1/(\phi\gamma)(w'(1) - p - \lambda k)$.



(a) Figure 4a (b) Figure 4b (c) Figure 4c

Figure 4: Phase diagrams for the dynamic system derived from (17) with constraint (10). On the x-axis we have x_t, on the y-axis s_t. The full thick lines denote the $\dot{x}_t = 0$ and $\dot{s}_t = 0$ phase curves, the orange and green lines show the stable manifolds. We assume $w(x) = ax$. In Figure 3a we use the parameters $a = 4$, $p = 1.5$, $k = 0.2$, $\gamma = 20$, $\phi = 0.106$. In Figure 3b we use the parameters $a = 4$, $p = 1.5$, $k = 0.2$, $\gamma = 20$, $\phi = 0.2$. In Figure 3c we use the parameters $a = 3.3$, $p = 1.5$, $k = 0.2$, $\gamma = 20$, $\phi = 0.2$. .

6 Conclusion

The literature on the private provision of public goods has mostly settled on social norms as a reason for which private agents would provide public goods. In this literature it has also been emphasized that there is room for public policy to induce the 'good' social norm of everyone contributing. As a result, the literature has to a large extent focused on whether or not public policy crowds in or out private provisions. In this article we have argued that it is not enough to focus on crowding in or out of private provisions as an argument for or against public policy. Instead, we argued that public policy needs to be assessed on the grounds of whether or not it is actually optimal from a society's perspective.

In order to study this we extended the model developed in Rege (2004) and introduced endogenous public policy. We showed that in very simple settings where the public policy is subject to a linear deadweight loss then it is not optimal to induce everyone to adhere to the social norm. Furthermore, we have shown that a Most Rapid Approach Path is the optimal solution and thus convergence and speed of convergence depends on the bounds of the public policy. In other words, results depend on in how far the policy maker can subsidize or tax private contributors. Optimality of the corner or interior solutions in the social norm then naturally depend on a variety of parameters.

Extending this simple model of a social norm to the case of non-linear deadweight losses turns out to have surprisingly complicated dynamics once a policy maker wants to take optimal policy into account. Here, we find a variety of potential outcomes, from a case with no interior equilibrium being optimal, to one with only a unique interior equilibrium, Skiba points and traps in social norms. Furthermore, while the equilibrium where everyone fully adheres to the social norm is always a potential equilibrium and it is, in fact, always locally stable, it does not need to be the optimal equilibrium.

In practical terms this result suggests that it is important to investigate the social optimality of government interventions in social norms. This has significant implications for example for the fashionable nudging (Thaler and Sunstein, 2008), for the analysis of social norms and public policy, and for in how far the government should intervene when it comes to the private provision of public goods. Furthermore, the analytical results on multiple steady states and various dynamics already show that the practical difficulties of judging the optimal policies could be very large. This certainly points to a stringent research agenda.

There are many applications and extensions that come to mind. Practically one would, for example, expect that a policy maker could be ignorant of a social norm that evolves through society. This case would correspond to one with asymmetric or limited information on the policy maker's side. The main issue would then be that a policy maker, oblivious to the fact that there is a social norm, would nevertheless set a certain policy, but only understand over time that his criterion was false. This could have important repercussions for the evolution of the norm, which could potentially not only crowd out the social norm but additionally result in sub-optimal policy decisions. The question would be whether one could design optimal policy rules despite having limited information on a social norm.

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A Linear cost

In this section we derive the Euler equation (12) using standard Hamiltonian techniques in order to provide intuition for the result.

The objective is to maximize welfare given by (9) with respect to s_t subject to $\dot{x}_t = x_t(1-x_t)\Delta U(x_t, s_t)$ and with bounds on subsidies in the form of $\underline{s} \leq s_t \leq \bar{s}$. The Hamiltonian is

$$\mathcal{H}(x, s, \mu) = V(x) - \gamma x s + \mu x(1-x)\Delta U(x, s).$$

the derivative with respect to s is $\mathcal{H}_s = -\gamma x + \mu x(1-x)$, so that

$$s = \begin{cases} \bar{s} & \text{if } \mu(1-x) > \gamma \\ \underline{s} & \text{if } \mu(1-x) < \gamma \\ \textit{indeterminate} & \text{if } \mu(1-x) = \gamma \end{cases} \tag{21}$$

the evolution of μ is given by

$$\begin{aligned}
\dot{\mu} = \phi\mu - \mathcal{H}_x &= \mu\phi - \mu\big[(1-2x)\Delta U + x(1-x)\Delta U_x\big] - \big(V'(x) - \gamma s\big) \\
&= \mu\phi - \mu\big[(1-2x)\Delta U + x(1-x)\Delta U_x\big] - V'(x) + \gamma\big[\Delta U - x\Delta U_x + (p - \lambda k)\big] \\
&= \mu\phi + [\gamma - \mu(1-2x)]\Delta U - [\gamma + \mu(1-x)]x\Delta U_x - V'(x) + \gamma(p - \lambda k)
\end{aligned}$$

if $s \in (\underline{s}, \bar{s})$ then $\mu(1-x) = \gamma$ and $\dot{\mu} = \gamma \dot{x}/(1-x)^2 = \gamma x \Delta U/(1-x)$ injecting this in the above equation gives

$$\gamma \frac{x}{(1-x)} \Delta U = \frac{\gamma \phi}{1-x} + \frac{\gamma}{1-x} \Big[x \Delta U - 2(1-x) \Delta U_x \Big] - V'(x) + \gamma(p - \lambda k)$$

that is $V'(x) - \gamma \big[p - \lambda k - 2\lambda(1-2k)x \big] = \frac{\gamma \phi}{1-x}$.

B Square costs

We write the Hamiltonian as

$$\mathcal{H} = V(x) - \gamma x s^2/2 + \mu x(1-x) \Delta U(x, s).$$

First order conditions yield

$$s = \frac{\mu(1-x)}{\gamma}, \tag{22}$$

$$\dot{\mu} = \mu \Big[\phi - (1-2x)\big(s - p + \lambda k\big) - (2-3x)\lambda(1-2k)x \Big] - [V'(x) - \gamma s^2/2] \tag{23}$$

Differentiating equation (22) wrt time and solving for $\dot{\mu}$ yields

$$\dot{\mu} = \frac{\gamma}{1-x} \dot{s} + \frac{\gamma s}{(1-x)^2} \dot{x}.$$

We now obtain a system of differential equations in $\{x, s\}$, which is given by

$$\dot{x} = x(1-x)\Delta U \tag{24}$$

$$\dot{s} = s\phi - s(1-x)[\Delta U + x \Delta U_x] - \frac{1-x}{\gamma} \Big[V'(x) - \frac{\gamma}{2} s^2 \Big] \tag{25}$$

The elements of the Jacobian matrix are

$$J_{xx} = (1-2x)\Delta U + x(1-x)\Delta U_x = (1-x)[\Delta U + x\Delta U_x] - x\Delta U \tag{26}$$

$$J_{xs} = x(1-x) \tag{27}$$

$$\begin{aligned} J_{ss} &= \phi - (1-x)[\Delta U + x\Delta U_x] - s(1-x) - (1-x)\frac{W_{xs}}{\gamma} \\ &= \phi - (1-x)[\Delta U + x\Delta U_x] \end{aligned} \tag{28}$$

$$\begin{aligned} J_{sx} &= s[\Delta U + x\Delta U_x] - s(1-x)[2\Delta U_x + x\Delta U_{xx}] + \frac{1}{\gamma}[(V' - \gamma s^2/2) - (1-x)V\prime\prime] \\ &= s[\Delta U + (3x-2)\Delta U_x] + \frac{1}{\gamma}[(V' - \gamma s^2/2) - (1-x)V\prime\prime] \end{aligned} \tag{29}$$

There are three candidates for steady states:

candidate 1: $x = 0$

If there is an optimal trajectory that converges toward $x = 0$, the associated s is bounded and converges toward the solution of the quadratic equation

$$0 = s\big(\phi - (s - p + \lambda k)\big) + \frac{1}{\gamma}\left(\gamma s^2/2 - \lambda k - w'(0) + p\right),$$

- There is a solution to this equation iff

$$w'(0) < \frac{\gamma}{2}\big(p - \lambda k + \phi\big)^2 + p - \lambda k.$$

- If $w'(0) < \frac{\gamma}{2}\big(p - \lambda k + \phi\big)^2 + p - \lambda k$, the two solutions are

$$s_{0\pm} = \phi + p - \lambda k \pm \sqrt{(\phi + p - \lambda k)^2 - \frac{2}{\gamma}\left[w'(0) - (p - \lambda k)\right]}$$

- For a solution s, the associated Jacobian matrix is lower triangular ($J_{xs} = 0$), the two eigenvalues are $\Delta U(0, s) = s - (p - \lambda k)$ and $\phi - \Delta U(0, s) = \phi + (p - \lambda k) - s$. There is a saddle path if $\Delta U(0, s) < 0$ or $\Delta U(0, s) > \phi$.

- s_{0-} is the only candidate, it is a saddle if $\Delta U(0, s_{0-}) < 0$ that is $s_{0-} < p - \lambda k$ or

$$\phi < \sqrt{(\phi + p - \lambda k)^2 - \frac{2}{\gamma}\left(w'(0) - (p - \lambda k)\right)}$$

or

$$\frac{2}{\gamma}\Big[w'(0) - (p - \lambda k)\Big] < (p - \lambda k)\Big[2\phi + (p - \lambda k)\Big]$$

– s_{0+} cannot be the limit of an optimal trajectory: s_{0+} is strictly larger than $p - \lambda k$, so that $\Delta U(x, s_{0+}) > 0$ in a neighborhood of 0. Along an optimal trajectory converging toward $(s_{0+}, x = 0)$, x_t is eventually decreasing (since $x_t > 0$) so $\Delta U(x_t, s_t)$ is eventually negative, a contradiction.

– Then $\Delta U(0, s_{0-}) < \phi$ and $(x = 0, s = s_{0-})$ is a saddle if and only if $\Delta U(0, s_{0-}) < 0$.

candidate 2: $x = 1$

- Substituting the $x_t = 1$ solution into the dynamic system yields the logical optimal solution $s_t = 0$.
- The Jacobian around the $\{x_2, s_2\} = \{1, 0\}$ steady state is given by

$$\mathcal{J}\Big|_{(x_2,s_2)} = \begin{bmatrix} p - (1-k)\lambda & 0 \\ \frac{w'(1)-p-k\lambda}{\gamma} & \phi \end{bmatrix}.$$

- As this is a lower triangular matrix we have that the eigenvalues are given by $EV_1 = p - (1-k)\lambda$ and $EV_2 = \phi$.
- This steady state is saddle path stable if $p < (1-k)\lambda$, which applies given Assumption 1.
- If $p < (1-k)\lambda$, in a neighborhood of $x = 1$ it is optimal to go toward $x = 1$.

candidate 3: $x = \frac{p-s-\lambda k}{(1-2k)\lambda}$, then x is solution of equation (20). There can be multiple solutions of this equation. The Jacobian at a solution is given by equations (26) to (28). The trace is equal to ϕ, the eigenvalues are:

$$EV_{\pm} = \frac{1}{2}\left[tr \pm \sqrt{tr^2 - 4D}\right]$$

in which D is the determinant of the Jacobian.

- If $D < 0$ there is a saddle equilibrium which is then possibly locally associated to an optimal trajectory.
- If $D > 0$ the equilibrium is unstable, with spirals if $D > \phi^2/4$.

The determinant is

$$D = x(1-x)\left\{\Delta U_x[\phi - (1-x)x\Delta U_x] - s(3x-2)\Delta U_x + [V' - \gamma s^2/2 - (1-x)V'']/\gamma\right\} \tag{30}$$

with $\Delta U_x = \lambda(1-2k)$ is positive, and decreasing with k. Note that $x(1-x)$ is maximized at $x = 1/2$ for a value of $1/4$; $3x-2$ is negative for $x < 2/3$.