

The endogenous formation of an
environmental culture
version 1

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Abstract

We develop an overlapping generations model with environmental quality and endogenous environmental culture. Based upon empirical evidence, preferences over culturally-weighted consumption and environmental quality are assumed to follow a Leontieff function. We find that four different regimes may be possible, with interior or corner solutions in investments in environmental culture and maintenance. Depending on the parameter conditions, there exists one of two possible, asymptotically stable steady states, one with and one without investments in environmental culture.

For low wealth levels, society is unable to free resources for environmental culture. In this case, society will only invest in environmental maintenance if environmental quality is sufficiently low. Once society has reached a certain level of economic development, then it may optimally invest a part of its wealth in developing an environmental culture. Environmental culture has not only a positive impact on environmental quality through lower levels of consumption, but it improves the environment through maintenance expenditure for wealth-environment combinations at which, in a restricted model without environmental culture, no maintenance would be undertaken. Environmental culture leads to a society with a higher indirect utility at steady state in comparison to the restricted model.

Our model leads us to the conclusion that, by raising the importance of environmental quality for utility, environmental culture leads to lower steady state levels of consumption and wealth, but higher environmental quality. Thus, for societies trapped in a situation with low environmental quality, investments in culture may induce positive feedback loops, where more culture raises environmental quality which in turn raises environmental culture. We also discuss how environmental culture may lead to an Environmental Kuznets Curve.

Keywords: environmental culture; overlapping generations model; environment; endogenous preferences.

JEL classification: Q56; D90.

1 Introduction

The environmental economics literature has extensively studied the role of consumption and abatement decisions for the interplay between economic growth and environmental quality. The focus of this literature has mostly been on the role of technical change, the limits to economic growth (Stokey 1998) imposed by resource constraints (Solow 1974, Dasgupta and Heal 1974), the usefulness of taxes (Jaffe et al. 2003) or educational measures (Prieur and Bréchet 2013). However, in studying these aspects, the literature has barely taken into account cultural aspects (Throsby 2000). Culture plays the important role of shaping the way in which our preferences are directed towards the environment. Specifically, Linton (1963, p.466) defines culture as being “the sum total of the knowledge, attitudes and habitual behavior patterns shared and transmitted by the members of a particular society.” Consequently, it also affects how we trade off considerations of economic growth and environmental quality. As a result, culture is not a static concept but develops endogenously. We focus here on the type of culture that affects society’s attitude towards the consumption-environment trade-off. We dub this particular type of culture the ‘environmental culture’. With the specific topic of environmental culture in mind, we have witnessed remarkable changes in attitudes towards the environment during the past few years. To understand how precisely these changes in attitude affect the consumption-environment trade-off, and the drivers of the social changes, we here study the interplay between environmental culture, economic decisions and the environment.

The first extension in this article is that environmental culture is endogenously determined by the agent. The modeling approach shares ideas from Rapoport and Vidal (2007) and John and Pecchenino (1994). In this framework, culture affects the how society values consumption when young relative to environmental quality when old. Examples for this are educational measures¹, the time spent on learning to appreciate the environment, or a social

¹For early empirical evidence see Van Liere and Dunlap (1980) or, more recently,

norm directed towards sustainability. Since these do not come for free, the model is based on the assumption that the choice of culture is costly. We discuss this more closely in section 2.

The second extension is that the agents' preferences are described by a Leontieff function. This implies that environmental quality and consumption are perfect complements and thus decisions follow a lexicographical order. It allows to place a stronger emphasis on the fact that, firstly, there is only a very limited trade-off between consumption and environment possible. This assumption is akin to the strong sustainability paradigm (see e.g. Daly 1992). Secondly, it suggests that agents place absolute priority on the factor of utility that is most needed. We discuss this further in section 2. Environmental culture is then modeled as a sort of mediator between the two, meaning that it acts as a weight on the relative valuation of consumption and the environment. Thus, in a materialistic society, environmental culture is likely to bear little influence and a unit of consumption is valued more highly than environmental quality. In contrast, an ecocentric society is characterized by a high level of environmental culture and environmental quality will receive a much higher weight in utility.

The results of this paper are as follows. For low wealth levels, society is unable to free resources for environmental culture. In this case, society will only invest in environmental maintenance if environmental quality is sufficiently low. Once society has reached a certain level of economic development, then it may optimally invest a part of its wealth in developing an environmental culture. When environmental quality and wealth are both sufficiently large, then society may find it optimal to temporarily over-invest in environmental culture. This is optimal until environmental quality is decreased to a level from which onwards it is important for society to also invest in maintenance. In other words, if there is no urgent need for society to improve environmental quality, then society will either invest in environ-

Franzen and Meyer (2010).

mental culture if it can afford to do so, or not invest in case it is too poor. Then, since investments in environmental culture raise the importance of environmental quality for utility, it becomes also more worthwhile for society to spend money on maintenance due to a positive feedback between culture and the environment. As a result, environmental culture has not only a positive impact on environmental quality through lower levels of consumption, but in addition it improves the environment through maintenance expenditure for wealth-environment combinations at which, in a restricted model without environmental culture, no maintenance would be undertaken. Technological improvements in emission or abatement efficiency both raise steady state wealth and environmental quality.

Our model leads us to the conclusion that by raising the importance of environmental quality for utility, environmental culture leads to lower steady state levels of consumption and wealth, but a higher environmental quality. Furthermore, indirect utility at steady state is higher when societies may invest in environmental culture. Basically, environmental culture not only helps to further appreciate environmental quality, but in addition it reduces consumption and thereby increases environmental quality. Thus, for sufficiently rich societies that are trapped in a steady state with low environmental quality, investments in environmental culture may induce positive feedback loops, where a higher appreciation of environmental quality due to environmental culture also induces society to improve environmental quality, which again drives increases in environmental culture.

The approach presented in this paper is closely related to the literature on endogenous preferences as well as social norms.² However, while a majority of the articles in this line of literature studies predetermined preferences, we develop a model of fully endogenous preferences.³ The article, furthermore,

²See e.g. Bowles (1998) for a general discussion. Other research includes e.g. endogenous discounting (Becker and Mulligan 1997, Schumacher 2009b), endogenous preferences from religious or group characteristics (Escriche et al. 2004), evolutionary selection or cultural traits (Bisin and Verdier 1998, Bisin and Verdier 2001, Hauk and Saez-Marti 2002).

³Predetermined preferences means that, although preferences are endogenously deter-

adds to the line of literature on cultural economics (Throsby 2000, Bowles 1998). Specifically, we here combine the major types of capital, namely physical capital, natural capital and cultural capital in one model.⁴ Our contribution is to study the endogenous formation of an environmental cultural capital. We think about this as an intangible form of capital, which defines the common ideas, beliefs and social norms and values of society. In this respect, there is not the usual environmental economics trade-off, where more physical capital may be substituted for less natural capital in production. In our model, environmental culture affects the relative valuation of environmental quality versus consumption, and thus physical capital.

The article is structured as follows. In section 2 we discuss more about the need to model environmental culture and why the choice of lexicographical preferences is useful in this setting. Section 3 introduces the model, section 3.1 solves the temporal equilibrium, and in section 3.2 we solve an explicit version that allows us to derive the global dynamics for the intertemporal equilibrium. In section 4 we discuss several potential concerns of robustness. Finally, section 5 concludes.

2 Motivation

In this section we introduce the reader to the concept of an environmental culture, and also discuss more fully our reasons for relying on lexicographical preferences in the model that we introduce below.

2.1 Environmental culture

The anthropologist Linton describes culture as "the sum total of the knowledge, attitudes and habitual behavior patterns shared and transmitted by

mined in the model, society cannot choose its preferences itself.

⁴For simplicity we neglect from human capital, see Becker (2009). This could be a worthwhile extension for future work.

the members of a particular society... Cultures are adaptive mechanisms and as such represent a response to the needs of our species” (Linton 1963, p.466). Another known definition is given in Altman and Chemers (1980), who describe culture as consisting of “... beliefs and perceptions, values and norms, customs and behaviors of a group or society.” Similar definitions by other anthropologists can be found in Tylor (1958) or Geertz (1957). In economics, culture has been most strongly associated with institutions (e.g. La Porta et al. 1997, Bowles 1998, Platteau 2000), social capital (Putnam and Leonardi 1993) and norms (Bisin and Verdier 2005). Through these, culture has significant potential to affect the environment. It thus goes without saying that culture plays a predominant role in shaping how we view, value and subsequently treat trade-offs that affect the environment. We call the specific type of culture that is associated with how mankind treats the planet earth the *environmental culture*.

One question is really what shapes an environmental culture. In general, the development of an environmental culture may arise in response to both the desire to orient oneself closer to nature, or due to the need to do so. Several articles have studied aspects of an environmental culture and how these affect the relationship between wealth, resource exploitation and the environment. Specifically, Sethi and Somanathan (1996) investigate endogenous social norms in a local common-property resource game. They find that two possible Nash-game equilibria may be stable, one being composed of an individualistic society, the other of a norm-guided society. Brekke et al. (2003) find, in a public good model with social norms directed to effort, that, despite allowing for social norms, this still leads to an underprovision of the public good. Nyborg et al. (2006) add replicator dynamics to the model of Brekke et al. (2003). This results in an equilibrium where either everyone acts according to the green norm, or everyone acts brown. Their social norm rests on the assumption that the moral motivation to act green is large if sufficiently many people act green, whereas the moral motivation

is small otherwise. In contrast, Schumacher (2009a) assumes that more people turn environmentalists, or adopt a green culture, when pollution is high, while preferences get directed less towards the environment when pollution is low. The common thread in these articles is that the level of the environmental culture is a direct response to social needs. These responses, and consequently the changes to the social norms, are assumed to be costless. Nevertheless, there exists ample evidence suggesting that an environmental culture develops in response to educational measures, pressure groups, or simply through spending more time in nature itself.

In this article we are assuming that the development of an environmental culture is costly. One way through which an environmental culture may be developed is through educational measures. Already in 1977, the UNESCO/UNEP held a conference on the need for environmental education, which resulted in the Tbilisi Declaration. In this document, it was suggested that (p. 13-14) “...adopting a holistic approach, rooted in a broad interdisciplinary base, [environmental education] recreates an overall perspective which acknowledges the fact that natural environment and man-made environment are profoundly interdependent.” Furthermore, emphasis was placed on the point that environmental education should (p. 14) “... encourage those ethical, economic and esthetic values which, constituting the basis of self-discipline, will further the development of conduct and improvement of the environment.”

For example, a consumption-oriented, materialistic society concerns itself very little with the effect of economic decisions on the environment. Instead, it places consumption in the forefront of decisions and neglects externalities imposed on the environment. In contrast, an ecocentric society is one that has a more holistic approach and incorporates environmental considerations into the decision-taking process. Consequently, the Tbilisi Declaration thrives in making society more ecocentric, by increasing the emphasis that society places on environmental issues relative to materialistic, consumption-

oriented ones. In order to achieve this, several objectives were defined, as summarized in Hungerford and Volk (1990). These are the building of awareness, sensitivity, attitudes, skills and participation. One of the main variables identified in that article and the meta-study covering 128 articles of Hines et al. (1987) as being a decisive, prerequisite component in an environmental behavior model is environmental sensitivity.

We here assume that the expenditure necessary to achieve a level of environmental culture X_{t+1} is given by $x_t > 0$. Function $\psi(x_t) > 0$, $\psi'(x_t) > 0$ then transforms one unit of expenditure x_t into $\psi(x_t)$ units of environmental culture. We can then denote the equation that describes the evolution of environmental culture by

$$X_{t+1} = \psi(x_t).$$

What we suggest here is that environmental culture depreciates fully during the course of one generation. Specifically, this rests on the assumption that a new generation does not ‘inherit’ any environmental culture from the previous one, and all educational measures or transfers of educational culture are costly. This assumption makes sense if one believes that environmental culture is not freely adopted through a parent-child relationship⁵, but, instead, is learned through costly educational measures that shape individuals’ attitudes, skills and participation. However, these costs may also arise through outdoor learning via e.g. field trips, through information provision (e.g. eco-labels), or social norms that induce inefficiencies which we, for simplicity, denote in monetary terms.

⁵There is, nevertheless, also evidence for an intergenerational transmission of preferences, see e.g. Bisin and Verdier (2005) or Schumacher (2009a) in an environmental economics setting. We discuss the implication of a costless intergenerational transmission of an environmental culture in section 4.1.1.

2.2 Preference structure

In general, economists assume that society can trade off consumption and environmental quality in their utility function along a smooth isoquant. This implies that, for example, society may stay on the same utility isoquant if it gives up a little environmental quality to gain a little more of consumption. Nevertheless, this trade-off very often does not, or cannot, happen. One reason for this is that society allocates resources or chooses according to a hierarchy of needs.⁶ In economics, this hierarchy of needs can be modeled through lexicographical preferences. As a consequence, consumption and environmental quality in the utility can not be easily substituted (Georgescu-Roegen 1954).⁷

In terms of empirical evidence, Diekmann and Franzen (1999) have shown that poorer societies rank environmental problems lower in comparison to other problems, while they view the severity of these problems similarly to rich societies. Consequently, societies may rank social decisions according to their needs, and address them mostly consecutively. A poor country may, therefore, place a stronger emphasis on consumption during its development stage, while a sufficiently rich country may have enough financial resources left over to treat the more pressing environmental problems. Only those societies that are at the same time sufficiently rich and have a satisfactory level of environmental quality may actually be able to trade off the two in the conventional sense of economic trade-offs. Other empirical studies have shown that around a quarter of individuals apply lexicographical preferences

⁶This has been suggested by Maslow's hierarchy of needs.

⁷This assumption on the utility function is also an intermediate position to the debate on strong versus weak sustainability. Brekke (1997) suggests that "[a] development is ... said to be weakly sustainable if the development is non-diminishing from generation to generation." In contrast, strong sustainability requires that both human and natural capital are kept intact (Brekke et al. 1997, Daly 1992). While strong sustainability is a requirement that is nearly impossible to be met for a developing society, weak sustainability may induce unsustainable levels of environmental quality. Instead, lexicographical preferences place attention on the input to utility that society ranks as being most crucial.

when it comes to wildlife preservation (Stevens et al. 1991) or to wetland preservation (Spash 2000).

We, thus, start off by assuming that society allocates resources based on a lexicographical utility function that takes the form $U = \min\{c, E\}$, where c is consumption and E environmental quality. However, a unit of consumption is generally not valued in the same way as a unit of environmental quality. In effect, as we argued above, the predominant environmental culture at the time of the decision-taking may strongly affect the relative valuation of consumption and environmental quality. This point is closely related to the arguments in Lockwood (1996), Gowdy (1997) or Spash (2000). Consequently, we take it that environmental culture affects the relative valuation of consumption versus environmental quality by assuming it to be a multiplicative factor for consumption. We, thus, obtain

$$\min\{X(x)c, E\}$$

As a result, the function $\min\{X(0)c, E\}$ is the basic preference of a society in which no expenditure towards an environmental culture is undertaken. A unit of environmental quality would then be worth $X(0)$ units of consumption. Thus, if $X(x)$ is sufficiently low, e.g. tending to zero, then consumption would obtain absolute priority over environmental quality, as long as the environment is not destroyed (i.e. goes to zero). The larger is $X(x)$, the more important becomes environmental quality to society. For example, assume $X(0) = 0.2$. In this case, five units of consumption are effectively viewed equivalent to one unit of environmental quality. This, thus, can be interpreted as a consumption-oriented society. An agent faced with high environmental quality is then likely to turn all his attention towards consumption as the minimum of his culture-weighted consumption compared to environmental quality is that what decides over his utility. Assume now that the agent invests $x > 0$ into environmental culture, such that $X(x) = 1$. Then one unit of consumption produces the same amount of utility as one unit of envi-

ronmental quality. In other words, environmental quality becomes relatively more important for utility than before.

3 The Model

In terms of motivation for the basic modeling structure we follow the convincing approach in John and Pecchenino (John and Pecchenino 1994). Thus, we assume that society's decisions are taken by a representative agent, who may raise taxes to finance environmental maintenance (abatement) or to increase environmental culture. This agent thus optimally allocates income towards maintaining environmental quality, improving environmental culture, or increasing savings for consumption when old.

Environmental quality

Environmental quality E_{t+1} represents the state of nature and is reduced by emissions that come from consumption, c_t , while it is increased through abatement, a_t . The law of motion is given by

$$E_{t+1} = g(E_t, c_t) + \gamma a_t, \quad (1)$$

where $\frac{\partial g(E_t, c_t)}{\partial E_t} \geq 0$, and $\frac{\partial g(E_t, c_t)}{\partial c_t} < 0$. We interpret E_{t+1} as the state of nature that the generation born at time t inherits. The coefficient $\gamma > 0$ on abatement defines how effective a unit of abatement is in improving environmental quality.

Environmental culture

We assume that environmental culture at time t is represented by X_t , increased by investments x_t through a production function $\psi(x_t)$ but depreciates fully during the course of each generation. The production function is assumed to have the following shape.

A 1 *Function* $\psi(x_t) > 0$, $\psi'(x_t) > 0$, $\psi''(x_t) \leq 0$.

As a result, environmental culture is set according to

$$X_{t+1} = \psi(x_t). \quad (2)$$

The assumption of full depreciation is justified based on the arguments in the previous section.

The agent

We assume that in each period there exist two generations, a young one and an old one. Each generation is represented by a single agent that lives for two periods, called young in the first period of life and old in the second period. We simplify by assuming away population growth. Young agents receive a labor income $w_t \geq 0$. They may spend this income either on capital formation $s_t \geq 0$ for consumption c_t when old, on abatement $a_t \geq 0$ or they may invest $x_t \geq 0$ in environmental culture. As old agents are not altruistic with respect to future generations they use their returns on saving for consumption only.

The utility function of the representative agent takes the form

$$u(X_{t+1}, c_{t+1}, E_{t+1}) = \min\{X_{t+1}c_{t+1}, E_{t+1}\}. \quad (3)$$

This specification, as explained in the previous section, rests on two assumptions. One, environmental quality and effective consumption are perfect complements. Two, the degree to which a unit of consumption is turned into effective consumption depends on the agent's investment in environmental culture.⁸

The constraints faced by the agent are as follows. He receives wages $w_t > 0$ which he can allocate to savings $s_t \geq 0$, to abatement $a_t \geq 0$, or to investing in environmental culture $x_t \geq 0$. Savings today receive an

⁸In this setup, it makes no difference whether we assume $X_t = \psi(x_t)$, or $X_{t+1} = \psi(x_t)$.

interest rate of $R_{t+1} > 1$, and the agent fully consumes his savings plus the interest obtained when old. In addition, he faces the law of motion for the environment, as given by equation (1).

$$w_t = s_t + x_t + a_t, \quad (4)$$

$$c_{t+1} = R_{t+1}s_t, \quad (5)$$

$$E_{t+1} = g(E_t, c_t) + \gamma a_t. \quad (6)$$

We call the equilibrium resulting from the assumptions and conditions above the temporal equilibrium. It is defined as follows.

Definition 1 *The temporal equilibrium consists of the allocations $\{s_t, a_t, x_t\}$, where at every $t = 0, 1, 2, \dots$, the young generation maximizes (3) subject to (4), (5) and (6), with w_t, E_t, R_{t+1} and c_t given.*

Lemma 1 $\exists x_t^m \geq 0$, such that $x_t = x_t^m$ solves

$$\max_{x_t \in [0, w_t]} \psi(x_t)R_{t+1}(w_t - x_t).$$

Proof of Lemma 1 *The derivative of function $\psi(x_t)R_{t+1}(w_t - x_t)$ with respect to x_t is $\psi'(x_t)R_{t+1}(w_t - x_t) - \psi(x_t)R_{t+1} \equiv \Theta(x_t)$. This function has the properties that $\Theta(0) = \psi'(0)R_{t+1}w_t - \psi(0)R_{t+1}$, while $\Theta(w_t) = -\psi(w_t)R_{t+1} < 0$, and $\Theta'(x_t) = \psi''(x_t)R_{t+1}(w_t - x_t) - 2\psi'(x_t)R_{t+1} < 0$. Consequently, an interior solution requires $w_t > \psi(0)/\psi'(0)$, in which case $x_t = x_t^m$, where x_t^m is the solution to $\Theta(x_t^m) = 0$. If $w_t \leq \psi(0)/\psi'(0)$, then there exists no interior solution to $\Theta(x_t^m) = 0$ and the optimal solution is $x_t^m = 0$. ■*

Thinking about the mathematical problem ex ante, then for interior choices the optimal solution would be to find a combination between x_t, a_t and s_t such that

$$E_{t+1} = \psi(x_t)c_{t+1}. \quad (7)$$

Re-writing this equation by substituting the constraints (4), (5) as well as the environmental law of motion (6) leads to

$$a_t = \frac{\psi(x_t)R_{t+1}(w_t - x_t) - g(E_t, c_t)}{\psi(x_t)R_{t+1} + \gamma}. \quad (8)$$

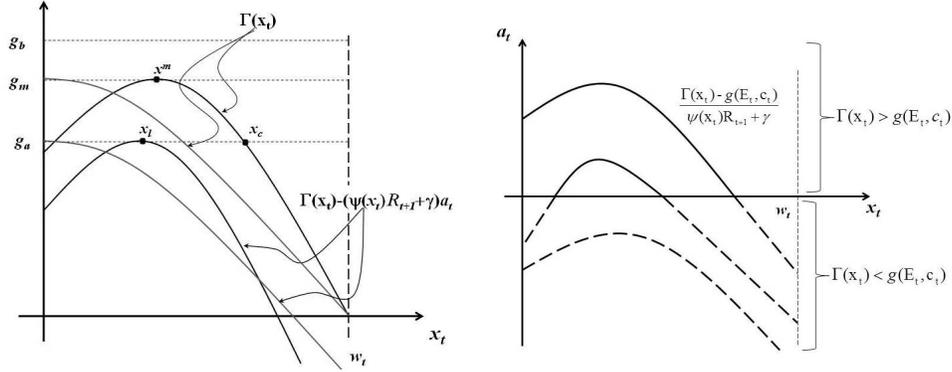
Here we have an equation of the two endogenous variables x_t and a_t that also implicitly determines s_t (via eq. (4)). We define

$$\Gamma(x_t) \equiv R_{t+1}\psi(x_t)(w_t - x_t) = X_{t+1}c_{t+1}. \quad (9)$$

Function $\Gamma(x_t)$ then represents the level of culturally-weighted consumption. In other words, the environmental culture transforms one unit of consumption in X_{t+1} units of environment. Function $\Gamma(x_t)$ can then be understood as potential, culture-weighted consumption. The interpretation of equation (8) is that abatement is positive as long as the potential, culture-weighted consumption level exceeds the given amount of environmental quality at the time at which the agent takes his choices.

For a given environmental quality at time t , $g(E_t, c_t)$, the agent will choose the optimal mix of a_t , s_t and x_t . Figure 1 illustrates how we can determine the optimal allocations starting from equation (7), (8) and (9). Intuitively speaking, an agent faced with a high level of environmental quality like g_b will be unable to increase his utility via investing in environmental quality since the level of the environment is already relatively high. Therefore, he will only maximize $\Gamma(x_t) = \psi(x_t)R_{t+1}(w_t - x_t)$. By Lemma 1 this maximum is given when $x_t = x_t^m$. Since he takes his income and interest rate as given, the maximization will simply be through $x_t \in [0, w_t]$. As the graph shows, for any g_t larger than or equal to $\Gamma(x_t^m)$, this point will be given by $x_t = x_t^m$. Clearly, x_t^m represents the optimal choice for x_t when $a_t = 0$. Obviously, if $\Gamma'(x_t) \leq 0$, $\forall x_t \in [0, w_t]$, then $x_t^m = 0$. Hence, a positive amount x_t will only decrease overall utility since the costs of increasing environmental culture outweigh the benefits. Or, in other words, the increase in environmental

Figure 1: Different optimal allocations



(a) Function $\Gamma(x_t)$ with corner and interior a_t (b) Potential allocations of a_t as a response to choices of x_t

culture costs the agent too much consumption, so that overall the culture-weighted consumption decreases. For $\Gamma'(0) > 0$, then the optimal $x_t^m > 0$. In this case, investments in environmental culture increase culture-weighted consumption.

In contrast, assume now that environmental quality at time t is low, at e.g. g_a . In this case keeping $a_t = 0$ would potentially waste income. If $\Gamma(x_t^m) > g(E_t, c_t)$, then a positive amount of abatement would increase utility. As the graph shows, the agent will have too much culture-weighted consumption (relative to environmental quality) for all $x_t < x_c$, while too little for all $x_t > x_c$. However, x_c is not an optimal allocation either as the agent can do better. He could invest less in environmental culture which would increase $X_{t+1}c_{t+1}$. The income saved can then be spent on abatement, which increases E_{t+1} by γa_t , which is the direct effect of higher abatement on environmental quality.

3.1 Solving the model

We now solve the model analytically. The agent's objective is to find the combination of s_t , x_t and a_t that leads to a maximum of utility given the constraints and initial conditions. By relying on equation (7) and substituting the constraints, we can write

$$E_{t+1} = \frac{R_{t+1}\psi(x_t)}{\gamma + \psi(x_t)R_{t+1}} \left(\gamma(w_t - x_t) + g(E_t, c_t) \right). \quad (10)$$

We use this as a new, reduced-form utility function,⁹ which we denote by

$$W(x_t) = \frac{\psi(x_t)}{\gamma + \psi(x_t)R_{t+1}} \left(\gamma(w_t - x_t) + g(E_t, c_t) \right). \quad (11)$$

The first-order condition¹⁰ of $W(x_t)$ with respect to x_t gives

$$\psi'(x_t) (\gamma(w_t - x_t) + g(E_t, c_t)) \leq \psi(x_t)(\gamma + \psi(x_t)R_{t+1}), \quad (12)$$

which holds with equality if $x_t > 0$. This condition states that, for interior solutions, a marginal increase in x_t will increase utility both through its effect of the cultural weight on consumption and on environmental quality. The right-hand side of equation (12) represents the marginal cost of the lower savings valued at the interest rate R_{t+1} and the cultural weight at which consumption and the environment are traded-off. In addition, the marginal cost of lower abatement a_t is given by γ and weighted by the environmental culture.

⁹The term R_{t+1} is equivalent to a monotone transformation, so we can neglect it.

¹⁰One can, equivalently, arrive at this result via maximizing $\psi(x_t)c_{t+1}$ subject to $\psi(x_t)c_{t+1} = E_{t+1}$. When substituting the constraints one can then solve for a_t and substitute this into $\psi(x_t)c_{t+1}$. This will lead to the same first-order condition as equation (12) below.

The second-order derivative is

$$W''(x_t) = (\psi''(x_t)(\gamma(w_t - x_t) + g_t) - 2\gamma\psi'(x_t))(\gamma + \psi(x_t)R_{t+1}) - 2\gamma\psi'(x_t)^2 R_{t+1}(\gamma(w_t - x_t) + g_t) < 0, \quad (13)$$

thus a maximum is assured.

We now define two bounds that are important for the latter analysis.

Definition 2 *We define the thresholds*

$$g_L \equiv \frac{\psi(0)}{\psi'(0)}(\gamma + \psi(0)R_{t+1}) - \gamma w_t,$$

and

$$g_H \equiv \frac{\psi(w_t)}{\psi'(w_t)}(\gamma + \psi(w_t)R_{t+1}).$$

As we shall show later, given Assumption 1 it is always true that $g_L < g_H$. Thus, g_L denotes a lower bound, while g_H defines an upper bound. We introduce now the temporal equilibrium.

In Proposition 1 we introduce the optimal choices of the representative agent given the maximization problem that we defined above.

Proposition 1 *At the temporal equilibrium, the model as described in equations (3) to (6) gives rise to at maximum four different regimes that arise depending on the following parameter conditions:*

REGIME 1. *If $g(E_t, c_t) \leq g_L \wedge g(E_t, c_t) < \psi(0)R_{t+1}w_t$, then there exists a unique $a_{1t}^* = \frac{\psi(0)R_{t+1}w_t - g(E_t, c_t)}{\gamma + \psi(0)R_{t+1}}$, $s_{1t}^* = w_t - a_{1t}^*$, and $x_{1t}^* = 0$.*

REGIME 2. *If $g(E_t, c_t) \geq \psi(0)R_{t+1}w_t \wedge w_t \leq \psi(0)/\psi'(0)$, then $a_{2t}^* = x_{2t}^* = 0$ and $s_{2t}^* = w_t$.*

REGIME 3. *If $g(E_t, c_t) \in (g_L, \Gamma(x_t^m)) \wedge w_t > \psi(0)/\psi'(0)$, then there exists a unique $x_{3t}^* \in (0, w_t)$ that solves equation (12), with $a_{3t}^* = \frac{\Gamma(x_{3t}^*) - g(E_t, c_t)}{\gamma + \psi(x_{3t}^*)R_{t+1}}$ and $s_{3t}^* = w_t - a_{3t}^* - x_{3t}^*$.*

REGIME 4. *If $\Gamma(x_t^m) \leq g(E_t, c_t) \wedge w_t > \psi(0)/\psi'(0)$, then $a_{4t} = 0$ and $s_{4t} = w_t - x_{4t}$, where x_{4t} is given by $x_{4t} = x_t^m$.*

Proof of Proposition 1 *See Proof of Proposition 1 in Appendix 1.*

The model here gives rise to a maximum of four different regimes. Which regime turns out to be the optimal one for the society then depends on the given economic and environmental conditions at the time of choice. For low wealth levels, society is unable to free resources for environmental culture. Furthermore, if environmental quality is sufficiently high, then society will find it optimal to simply direct all income towards savings and thus maximizes consumption (Regime 2). This is a result of the strict lexicographical preference ordering. Based on the maxi-min choice criterion, a relatively high level of environmental quality would make society waste income if it is directed towards increasing environmental quality. Consequently, income is spent on increasing the most needed component of utility, namely culture-weighted consumption, a result akin to Maslow's theory of needs. This Regime 2 emphasizes the zero maintenance, minimum culture (temporal) equilibrium as a result of a low wealth and high environmental quality combination.

In contrast, if environmental quality is relatively low and wealth is not sufficiently high (Regime 1), then society should also target its wealth towards investments in environmental quality. The choices undertaken by society in this regime are then very close to those that are obtained via a utility function that does not place so much emphasis on basic needs (c.f. John and Pecchenino 1994).

When environmental quality and wealth are both sufficiently high, then society may find it optimal to temporarily over-invest in environmental culture (Regime 4). This regime is characterized by an abundant environmental quality and wealth. This is optimal until environmental quality is decreased to a level from which onwards it is worthwhile for society to also invest in environmental quality (Regime 3). In other words, if there is no urgent need for society to improve environmental quality, then society will either invest in an environmental culture if it can afford to do so, or not invest in case

it is too poor. This result is related to the view of cultural ecology, as e.g. defined in Berry (1975). In this line of literature, it is emphasized that the environment at a given point in time determines both culture and behavior. While we show this to be true at a given point in time, we also find that environmental culture is able to affect future environmental quality. Thus, there exist feedback loops between both the environment and culture.

We now look at some comparative statics assuming we are in Regime 3, such that we have an interior solution to x_t and a_t . In this case, changes in the efficiency of maintenance lead to

$$\frac{dx_{3t}^*}{d\gamma} = \frac{\psi(x_{3t}^*) - \psi'(x_{3t}^*)(w_t - x_{3t}^*)}{\Omega} > 0,$$

where $\Omega = \psi''(x_{3t}^*)(\gamma(w_t - x_{3t}^*) + g_t) - 2\psi'(x_{3t}^*)(\gamma + \psi'(x_{3t}^*)R_{t+1}) < 0$. Based upon Lemma 1, it is easy to see that $x_{3t}^* < x_t^m$, and therefore $\psi(x_{3t}^*) < \psi'(x_{3t}^*)(w_t - x_{3t}^*)$. The denominator is the second-order condition (13) of the maximization problem (11) and is negative. Consequently, increases in the efficiency of maintenance lead to increases in the optimal expenditure on environmental culture.

In case the given level of environmental quality is larger, then investments in environmental culture react according to

$$\frac{dx_{3t}^*}{dg_t} = -\frac{\psi'(x_{3t}^*)}{\Omega} > 0.$$

Thus, for higher levels of environmental quality, and for lower consumption levels of the previous generation, the current generation will profit from increasing investments in environmental culture. This arises because a higher given level of environmental quality allows to increase the agent's indirect utility. Consequently, the agent will raise culture-weighted consumption, and one means to do so is via increases in environmental culture.

In addition, a wealthier agent will change his investments in environmen-

tal culture according to

$$\frac{dx_{3t}^*}{dw_t} = -\frac{\gamma\psi'(x_{3t}^*)}{\Omega} > 0.$$

Thus, he will increase environmental culture, and one reason is simply that a higher wealth frees more of the agent's income for investments in environmental culture.

As a remark, in the extreme case of $\psi(0) = 0$, the lower bound g_L will be given by $g_L = 0$. In this case, both Regimes 1 and 2 will not exist, and there will always be an interior solution in x_t . Consequently, only Regimes 3 and 4 apply. The existence of a corner solution in $x_t = 0$, and therefore whether or not society may optimally find itself in Regime 1 or 2, thus depends on whether or not there is a minimal, positive amount of environmental culture at each point in time.

We also note that the bounds identifying the four different regimes are time-varying. They are an implicit function of the interest rate, the wages, and, in the case of the bound $\Gamma(x_t^*)$ which we obtained from the first-order condition equation (12), also depend on the given environmental quality and consumption. Consequently, the regions that make up the four regimes depend on the level of economic development and the condition of the environment. How these bounds evolve over time, therefore, depends on the intergenerational structure of the model, which we now define.

3.2 The full model

In the previous section we derived the optimal decisions of a representative agent for a given return on capital and for given wages. Consequently, we could only analyze the effect of the agent's choices for a given point in time. We now extend the results in the previous section by assuming a law of motion for capital. In doing so we can derive the evolution of our economy over time and study its potential convergence to a steady state. In addition to

introducing how capital accumulates, we also provide an explicit functional for environmental quality and for the endogenous cultural weight, which will facilitate the subsequent analysis at a minimal loss of generality.

A 1 *Capital accumulation is given by $s_t = k_{t+1}$.*

Capital is assumed to depreciate fully within the course of one generation. This assumption is borrowed from de la Croix and Michel (2002). It approximately corresponds to estimates of the speed of capital depreciation during one generation.

A 2 *For all $k_t > 0$, the production function takes the form $f(k_t) = k_t^\alpha > 0$, $\alpha \in (0, 1)$.*

We assume a representative firm that produces under constant returns to scale with a production function satisfying the following assumptions. We normalize Total Factor Productivity to one for simplicity. Based on this explicit production function, the wage constraint and the law of motion for capital accumulation, then the maximum feasible, constant level of capital is given by $k^{\max} = (1 - \alpha)^{\frac{1}{1-\alpha}}$.¹¹

A 3 *For all $x_t > 0$, the culture function assumes $\psi(x_t) = \psi_0 + \psi_x x_t > 0$, with $\psi_0 > 0$ and $\psi_x > 0$.*

The environmental culture function thus starts at the value $\psi_0 > 0$, and increases linearly with investments in environmental culture. For example, a consumption-oriented society is likely to be characterized by a cultural weight less than one. In this case, a unit of environmental quality would be valued less than a unit of consumption. An investment in environmental culture leads to a marginal increase by $\psi_x > 0$.

¹¹In the subsequent analysis we shall concentrate on the case where $k_t \in [0, k^{\max}]$, although the results are also applicable outside of this domain for non-negative k_t .

A 4 For all $E_{t+1} > 0$, the environment evolves according to $E_{t+1} = E_t - \beta c_t + \gamma a_t$. For $E_t - \beta c_t + \gamma a_t \leq 0$, we assume the lower bound $E_{t+1} = 0$, $\forall \tau \geq t$.

where $\beta > 0$ denotes emissions per unit consumption and $\gamma > 0$ the effectiveness of abatement. This thus implies that $g(E_t, c_t) = E_t - \beta c_t$. On the relevant time-scale of mankind, the own regeneration rate of the environment is assumed to be negligible. Thus, in contrast to John and Pecchenino (1994), environmental quality does not return to its natural level in case there is no human interference. One could allow for a natural regeneration rate, as in e.g. Jouvét et al. (2005). However, our focus is on studying whether investments should be directed towards abatement or altruism, and therefore we want to simplify the theoretical framework as much as possible and necessary. We discuss the implication of this assumption in section 4.

A 5 We define $\eta \equiv (1 - \alpha)\gamma - \alpha\beta$ and assume that $\eta > 0$.

This last assumption assures an interior solution to some cases. We return to the implication of this assumption at a later stage.

We can now define the intertemporal equilibrium.

Definition 3 Given the capital stock k_0 and the environmental quality E_0 , an intertemporal equilibrium is a temporal equilibrium that furthermore satisfies, for all $t > 0$, the capital accumulation condition $k_{t+1} = s_t$.

We now look at the four cases identified in Proposition 1 separately and then combine the results together. Given our explicit functional forms as well as Assumptions 2 to 5 we obtain the following explicit forms for the

thresholds that we introduced above.

$$\begin{aligned}
g_L &= \frac{\psi_0}{\psi_x}(\gamma + \psi_0\alpha k_{t+1}^{\alpha-1}) - \gamma(1 - \alpha)k_t^\alpha, \\
g_H &= \frac{\psi_0 + \psi_x(1 - \alpha)k_t^\alpha}{\psi_x}(\gamma + (\psi_0 + \psi_x(1 - \alpha)k_t^\alpha)\alpha k_{t+1}^{\alpha-1}), \\
\Gamma(x_t^m) &= \alpha k_{t+1}^{\alpha-1} \frac{(\psi_x(1 - \alpha)k_t^\alpha + \psi_0)^2}{4\psi_x},
\end{aligned}$$

where threshold x_t^m is given by $x_t^m = \frac{\psi_x(1-\alpha)k_t^\alpha - \psi_0}{2\psi_x}$. $\Gamma(x_t^m)$ is only defined for $(1 - \alpha)k_t^\alpha \geq \frac{\psi_0}{\psi_x}$, which implies $k_t \geq \left[\frac{\psi_0}{(1-\alpha)\psi_x}\right]^{1/\alpha}$. We provide several analytical conditions that help us in deriving the subsequent results.

Lemma 2 *Given Assumptions 1 to 5 we find that $g_L > \psi(0)R_{t+1}w_t$ iff $w_t > \psi(0)/\psi'(0)$.*

Proof of Lemma 2 *Condition $g_L > \psi_0 R_{t+1} w_t$ can easily be re-written as $(\psi(0)R_{t+1} + \gamma)(\psi(0)/\psi'(0) - w_t) > 0$ and thus holds iff $w_t < \psi(0)/\psi'(0)$. ■*

Lemma 3 *Based on Assumptions 1 to 5 we obtain that $\Gamma(x_t^m) < g_H$.*

Proof of Lemma 3 *We substitute the explicit conditions into $\Gamma(x_t^m) < g_H$. Then we rewrite and simplify $\alpha k_{t+1}^{\alpha-1} \frac{(\psi_x(1-\alpha)k_t^\alpha + \psi_0)^2}{4\psi_x} < \frac{\psi_0 + \psi_x(1-\alpha)k_t^\alpha}{\psi_x}(\gamma + (\psi_0 + \psi_x(1-\alpha)k_t^\alpha)\alpha k_{t+1}^{\alpha-1})$. This holds always. ■*

This Lemma is quite obvious in the sense that the maximum value of $\Gamma(x_t)$ must be less than the upper threshold g_H since the upper threshold is derived for $x_t = w_t$.

Lemma 4 *Given Assumptions 1 to 5 we find that $\Gamma(x_t^m) > \psi(0)R_{t+1}w_t$.*

Proof of Lemma 4 *Substituting the explicit functional forms into condition $\Gamma(x_t^m) > \psi(0)R_{t+1}w_t$ and slightly simplifying gives $(\psi_0 + \psi_x(1 - \alpha)k_t^\alpha)^2 > 4\psi_0\psi_x(1 - \alpha)k_t^\alpha$. Expanding the square term, simplifying and placing all terms on one side gives $\psi_0^2 + \psi_x^2(1 - \alpha)^2 k_t^{2\alpha} - 2\psi_0\psi_x(1 - \alpha)k_t^\alpha > 0$. This can then be re-written as $(\psi_0 - \psi_x(1 - \alpha)k_t^\alpha)^2 > 0$, which holds always. ■*

Combining Lemma 3 and Lemma 4 we obtain $\Gamma(x^m) \in (\psi(0)R_{t+1}w_t, g_H)$. As $\Gamma(x_t)$ is maximized at $x_t = x_t^m$, then $\Gamma(x^*) \leq \Gamma(x^m)$.

For matters of comparison, we also define a restricted model

Definition 4 *The restricted model is given by equations (3) to (6) with $x_t = 0, \forall t$.*

Thus, the restricted model is simply the original one introduced above where the young generation cannot invest in environmental culture. As a result, in this model we will always have that $X_{t+1} = \psi(0)$.

A direct implication of Lemma 4 is that, for a sufficiently high level of environmental quality, environmental culture leads to earlier investments in maintenance expenditure in comparison to the restricted model. Intuitively, since investments in environmental culture raise the importance of environmental quality for utility, then it becomes also more worthwhile for society to invest in maintenance due to the positive feedback between culture and the environment. The capital-environment combination in which environmental culture leads to positive abatement expenditure in contrast to the restricted model where no abatement expenditure is undertaken is given by $\Gamma(x_{3t}^*) \in (\psi(0)R_{t+1}w_t, \Gamma(x_t^m))$. As a result, environmental culture has not only a positive impact on environmental quality through lower levels of consumption, but in addition it improves the environment through maintenance expenditure for capital-environment combinations at which, without environmental culture, no maintenance would be undertaken.

We are now in a position to study the dynamic evolution of the four regimes that we identified above.

3.2.1 Regime 1

If $g(E_t, c_t) \leq g_L \wedge g(E_t, c_t) < \psi(0)R_{t+1}w_t$, then there exists a unique $a_{1t}^* = \frac{\psi(0)R_{t+1}w_t - g(E_t, c_t)}{\gamma + \psi(0)R_{t+1}}$, $s_{1t}^* = w_t - a_{1t}^*$, and $x_{1t}^* = 0$.

This gives rise to the dynamic system

$$k_{t+1} = \frac{E_t + \eta k_t^\alpha}{\gamma + \psi_0 \alpha k_{t+1}^{\alpha-1}}, \quad (14)$$

$$E_{t+1} = \psi_0 \alpha k_{t+1}^{\alpha-1} \frac{E_t + \eta k_t^\alpha}{\gamma + \psi_0 \alpha k_{t+1}^{\alpha-1}}. \quad (15)$$

The steady state equations are given

$$E = \psi_0 \alpha k^\alpha \equiv z_1(k), \quad (16)$$

$$E = \gamma k + (\psi_0 \alpha - \eta) k^\alpha \equiv z_2(k). \quad (17)$$

The steady state in this regime can be characterized as follows.

Proposition 2 *In Regime 1 the unique steady state is given by $\{\bar{k}_1, \bar{E}_1\} = \{(\eta/\gamma)^{\frac{1}{1-\alpha}}, \alpha\psi_0(\eta/\gamma)^{\frac{\alpha}{1-\alpha}}\}$ and it is locally asymptotically stable. Convergence to the steady state is monotonic.*

Proof of Proposition 2 *See Proof of Proposition 2 in Appendix.*

At steady state $\{\bar{k}_1, \bar{E}_1\}$, condition $g_t \leq g_L$ implies $\bar{k}_1 \leq \psi_0/\psi_x$, which is equivalent to the parameter condition $(\eta/\gamma)^{\frac{1}{1-\alpha}} \leq \psi_0/\psi_x$, while condition $g_t < \psi(0)R_{t+1}w_t$ always holds.¹²

Regime 1 occurs if the given environmental quality at the time that the agent takes his decision is so low that investments in environmental culture would not be able to increase utility. However, a positive level of abatement would be worthwhile since an increase in environmental maintenance reduces the gap between environmental quality and culture-weighted consumption. This then leads to a convergence to a steady state without investments in environmental culture, but a positive amount of environmental maintenance. The economy will be caught in a low-culture trap, where abatement is undertaken to safeguard a minimum level of environmental quality.

¹²We can also easily show that, if \bar{k}_1 exists, then $k^{\max} > \bar{k}_1$. This applies since $k^{\max} = (1-\alpha)^{\frac{1}{1-\alpha}}$, and also $\bar{k}_1 = (\eta/\gamma)^{\frac{1}{1-\alpha}}$.

3.2.2 Regime 2

If $g(E_t, c_t) \geq \psi(0)R_{t+1}w_t \wedge w_t \leq \psi(0)/\psi'(0)$, then $a_{2t}^* = x_{2t}^* = 0$ and $s_{2t}^* = w_t$. We can easily show that $g_L \geq \psi(0)R_{t+1}w_t$ requires $w_t \leq \psi(0)/\psi'(0)$.

In this case the dynamic system reduces to

$$k_{t+1} = (1 - \alpha)k_t^\alpha, \quad (18)$$

$$E_{t+1} = E_t - \alpha\beta k_t^\alpha. \quad (19)$$

Clearly, no steady state $\{\bar{E}_2, \bar{k}_2\}$ exists in this case. The explicit solution to the difference system (18) can be written as

$$k_t = (1 - \alpha)^{\frac{1}{\alpha}} \left(\sum_{\tau=0}^t \frac{(1)_\tau \alpha^\tau}{\tau!} - 1 \right) k_0^{\alpha^t} \equiv \theta(k_0, t), \quad (20)$$

$$E_t = E_0 - \alpha\beta \sum_{\tau=0}^t \theta(k_0, \tau), \quad (21)$$

with $(1)_\tau$ being the Pochhammer symbol which denotes the falling factorial $(x)_y = x(x+1)\dots(x+y-1)$ (see Abramowitz and Stegun 1972).

This Regime 2 emphasizes zero maintenance and no investment in environmental culture as a result of two sets of conditions that are working together. The first set of conditions states that low wages with a too small investment sensitivity of environmental cultural leads to a corner solution in cultural investments. The second set of conditions states that investments in environmental maintenance do not pay off since the given level of environmental quality already exceeds effective consumption. With the laws of motion for capital accumulation and environmental quality in mind, we then see that environmental quality is only reduced (due to the zero maintenance), while maximum effort is directed towards capital accumulation. As a result, environmental quality is reduced at an increasing rate due to the growing levels of consumption. Consequently, this Regime 2 does not lead to an equilibrium. Instead, Regime 2 converges into either Regime 1 or Regime 4.

It converges into Regime 4 if capital accumulates faster than environmental quality is destroyed, which would, for example, be the case for a low emission rate (low β) or a high share of capital in production (high α).

3.2.3 Regime 3

If $g(E_t, c_t) \in (g_L, \Gamma(x_t^m)) \wedge w_t \geq \psi(0)/\psi'(0)$, then there exists a unique $x_{3t}^* \in (0, w_t)$ that solves equation (12), with $a_{3t}^* = \frac{\Gamma(x_{3t}^*) - g(E_t, c_t)}{\gamma + \psi(x_{3t}^*)R_{t+1}}$ and $s_{3t}^* = w_t - a_{3t}^* - x_{3t}^*$.

The equations that are describing the dynamic system in this case are given by

$$\eta k_t^\alpha + E_t - \gamma x_t = \frac{\psi_0 + \psi_x x_t}{\psi_x} (\gamma + (\psi_0 + \psi_x x_t) \alpha k_{t+1}^{\alpha-1}), \quad (22)$$

$$E_{t+1} = (\psi_0 + \psi_x x_t) \alpha k_{t+1}^\alpha, \quad (23)$$

$$k_{t+1} = (1 - \alpha) k_t^\alpha - x_t - a_t, \quad (24)$$

$$E_{t+1} = E_t - \alpha \beta k_t^\alpha + \gamma a_t. \quad (25)$$

Equation (22) comes from the first-order condition (12), equation (23) is equation (7) that derives directly from the maximin criterion, while equation (24) is the capital accumulation equation and (25) the law of motion for environmental quality. Solving equation (22) gives the interior solution¹³ for x_t by

$$x_{3t}^* = \frac{1}{\alpha \psi_x k_{t+1}^{\alpha-1}} \left[-(\gamma + k_{t+1}^{\alpha-1} \alpha \psi_0) + \sqrt{\gamma^2 + \alpha k_{t+1}^{\alpha-1} (\gamma \psi_0 + E_t \psi_x + \psi_x \eta k_t^\alpha)} \right], \quad (26)$$

$$\equiv \rho(E_t, k_t, k_{t+1}). \quad (27)$$

Condition $g_t > g_L$ assures that $x_3^* > 0$. Then we use equations (24) and (25) to substitute out a_t , (23) to substitute out E_{t+1} , and substitute the optimal solution $x_{3t}^* = \rho(E_t, k_t, k_{t+1})$. Thus, the system describing the

¹³We neglect the negative root.

dynamic evolution of case 3 is given by

$$E_{t+1} = (\psi_0 + \psi_x \rho(E_t, k_t, k_{t+1})) \alpha k_{t+1}^\alpha, \quad (28)$$

$$\begin{aligned} \psi_0 \alpha k_{t+1}^\alpha + \gamma k_{t+1} &= E_t + \eta k_t^\alpha \\ &\quad - (\psi_x \alpha k_{t+1}^\alpha + \gamma) \rho(E_t, k_t, k_{t+1}). \end{aligned} \quad (29)$$

As shown above, condition $g_L < g_H$ is always satisfied. Condition $g_t > g_L$ leads to $(\psi_x k_{t+1} - \psi_0)(\psi_0 \alpha k_{t+1}^{\alpha-1} + \gamma) > \psi_x (\psi_x \alpha k_{t+1}^\alpha + \gamma) \rho$. A necessary condition for $g_t > g_L$ to hold is thus $k_{t+1} > \psi_0 / \psi_x$. The time-constant versions of equations (28) and (29) are given¹⁴ by

$$E = \gamma \left(2k - \frac{\psi_0}{\psi_x} \right) + (\alpha k \psi_x - \eta) k^\alpha \equiv w_1(k), \quad (30)$$

$$\begin{aligned} E &= \frac{1}{2} \left[-2\gamma k + \alpha k^{\alpha+1} \psi_x \right. \\ &\quad \left. + \sqrt{k(k(\alpha \psi_x k^\alpha - 2\gamma)^2 + 4\alpha k^\alpha (\gamma \psi_0 + \eta \psi_x k^\alpha))} \right] \equiv w_2(k). \end{aligned} \quad (31)$$

The steady state is then obtained by combining the equations (22) to (25) and is given by

$$\begin{aligned} &\left(\gamma + \left(\psi_0 + \psi_x \left[\frac{\eta}{\gamma} k^\alpha - k \right] \right) \alpha k^{\alpha-1} \right) k \\ &= \frac{\psi_0 + \psi_x \left[\frac{\eta}{\gamma} k^\alpha - k \right]}{\psi_x} \left(\gamma + \left(\psi_0 + \psi_x \left[\frac{\eta}{\gamma} k^\alpha - k \right] \right) \alpha k^{\alpha-1} \right). \end{aligned} \quad (32)$$

Proposition 3 *There exists a unique steady state $\{\bar{k}_3, \bar{E}_3\}$ to the dynamic system (28) and (29) given by equation $2\psi_x \bar{k}_3 = \psi_0 + \psi_x \frac{\eta}{\gamma} \bar{k}_3^\alpha$ and $\bar{E}_3 = \alpha \bar{k}_3^{\alpha+1} \psi_x$, if $\bar{k}_3 > \psi_0 / \psi_x$. This steady state is locally asymptotically stable and approached monotonically.*

Proof of Proposition 3 *See Proof of Proposition 3 in Appendix.*

¹⁴For their derivation see Appendix 2.

Regime 3 is the only regime that sees an interior solution in environmental culture that actually converges to a steady state. In this regime, given environmental quality is sufficiently high so that investments in environmental culture make sense. At the same time, it is not high enough in order to induce a corner solution in maintenance. Or, in other words, the maximum potential level of culture-weighted consumption exceeds the current level of environmental quality, which gives some room for investments in environmental quality. These positive investments in environmental quality help to maintain a level of environmental quality at which it is worthwhile to also invest in environmental culture.

3.2.4 Regime 4

If $\Gamma(x_t^m) \leq g(E_t, c_t) \wedge w_t > \psi(0)/\psi'(0)$, then $a_{4t} = 0$ and $s_{4t} = w_t - x_{4t}$, where x_{4t} is given by $x_{4t} = x_t^m$.

Threshold x_t^m is given by $x_t^m = \frac{\psi_x(1-\alpha)k_t^\alpha - \psi_0}{2\psi_x}$, leading to the dynamic system

$$E_{t+1} = E_t - \alpha\beta k_t^\alpha, \quad (33)$$

$$k_{t+1} = \frac{\psi_x(1-\alpha)k_t^\alpha + \psi_0}{2\psi_x}. \quad (34)$$

This holds until either $g_t < \Gamma(x_t^m)$ or $\psi_x(1-\alpha)k_t^\alpha \leq \psi_0$. If $1-\alpha > (\psi_0/\psi_x)^{1-\alpha}$, then $k_{t+1} > (\leq)k_t$ for $k_t < (\geq)\frac{\psi_0 + \psi_x(1-\alpha)k_t^\alpha}{2\psi_x}$. Consequently, for $1-\alpha \geq (\psi_0/\psi_x)^{1-\alpha}$, then the optimal solutions for k_t and E_t will move from Regime 4 to Regime 3 over time. Instead, if $1-\alpha < (\psi_0/\psi_x)^{1-\alpha}$, then k_t and E_t converge either to Regime 3 or to Regime 2. We conclude that there exists no steady state in Regime 4.

In Regime 4, environmental quality is so high that it does not pay off to invest in maintenance. At the same time, income is sufficiently high for agents to spend a part of it on investments in environmental culture. As a result, environmental quality is diminished over time.

4 Discussion

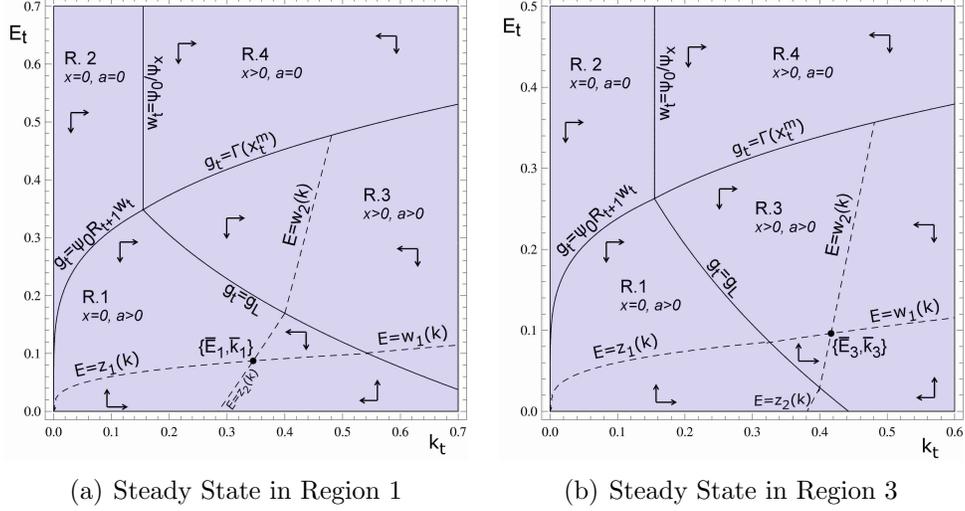
From the analysis above it is clear that either the steady state in Regime 1 gets picked up, or the one of Regime 3. Which one will be asymptotically approached depends on the parameter configurations. This is shown graphically in Figure 2 for two sets of generic parameter conditions. Under the first set of parameters ($\alpha = 0.3$, $\beta = 1.5$, $\gamma = 2$, $\psi_0 = 0.4$, $\psi_x = 1$), the steady state in Regime 1 gets selected, while under the second set of parameter conditions ($\alpha = 0.3$, $\beta = 1$, $\gamma = 2.2$, $\psi_0 = 0.4$, $\psi_x = 1$), it is the steady state in Regime 3 that is approached asymptotically. The analytical derivations for the general shape of the regions is available in Appendix 2. We denote the steady state curves of Regime 1 as $E = z_1(k)$ and $E = z_2(k)$, while those of Regime 3 as $E = w_1(k)$ and $E = w_2(k)$. They are depicted as the dashed lines in Figure 2. The boundaries of the four regions are defined according to the conditions in Proposition 1 together with the explicit functional forms and the law of motion for capital accumulation. Whether the dynamics in Regime 4 converge necessarily to Regime 3 or whether they may also converge to Regime 2 depends on whether condition $1 - \alpha \geq (\psi_0/\psi_x)^{1-\alpha}$ applies or not. If it applies, then k_t and E_t converge from Regime 4 into Regime 3, otherwise they may also converge to Regime 2. The arrows depict the dynamics for the case of $1 - \alpha \geq (\psi_0/\psi_x)^{1-\alpha}$.

We now clarify the role of the endogenous environmental culture further. Assume that the agent cannot invest in environmental culture. We are, thus, in the case of the *restricted model*. This implies that the optimal decision for abatement is

$$a_t = \frac{\psi(0)R_{t+1}w_t - g(E_t, c_t)}{\gamma + \psi(0)R_{t+1}},$$

if $\psi(0)R_{t+1}w_t > g(E_t, c_t)$, and $a_t = 0$ otherwise. Thus, in case the agent cannot invest in environmental culture, we still recover Regime 1 and 2, but these are not anymore constrained by the other two regimes. Using the

Figure 2: Different optimal allocations



explicit functional forms, this choice for a_t then leads to the dynamic system

$$E_{t+1} = \psi_0 \alpha k_{t+1}^\alpha, \quad (35)$$

$$k_{t+1} = \frac{E_t + \eta k_t^\alpha}{\gamma + \psi_0 \alpha k_{t+1}^{\alpha-1}}. \quad (36)$$

This dynamic system is exactly the same as the one arising in Regime 1, and consequently the steady state and the dynamics are equivalent. As a result, if the agent cannot invest in environmental culture, then he will necessarily pick up the steady state in Regime 1. If the parameter combinations induce the agent who may invest in environmental culture to also choose the steady state in Regime 1, then both cases will lead to the same outcome. However, the difference arises if the agent may freely invest in environment culture and parameter combinations make him pick up the steady state in Regime 3. In this case, the agent will invest in environmental culture, inducing a lower steady state level of capital, but a higher steady state level of environmental quality. This arises since the improvements in environmental culture raise

culture-weighted consumption, which induces the agent to reduce savings in favor of investments in environmental culture (and potentially abatement). The reduction in savings leads to a lower steady state capital stock, but also to less consumption and therefore higher steady state environmental quality. We summarize this in the following proposition. We denote the variables of the restricted model in which the young generation cannot invest in environmental culture with a hat.

Proposition 4 *If $g(\hat{E}_t, \hat{c}_t) < \psi(0)\hat{R}_{t+1}\hat{w}_t$, then $\hat{a}_t = \frac{\psi(0)\hat{R}_{t+1}\hat{w}_t - g(\hat{E}_t, \hat{c}_t)}{\gamma + \psi(0)\hat{R}_{t+1}}$, and $\hat{s}_t = \hat{w}_t - \hat{a}_t$.*

- *For $g(\hat{E}_t, \hat{c}_t) < \psi(0)\hat{R}_{t+1}\hat{w}_t$, then $\hat{k} = \bar{k}_1 = (\eta/\gamma)^{\frac{1}{1-\alpha}}$, and $\hat{E} = \bar{E}_1 = \psi_0\alpha\bar{k}_1^\alpha$.*
- *For $g(\hat{E}_t, \hat{c}_t) \geq \psi(0)\hat{R}_{t+1}\hat{w}_t$, then $\hat{k} > \bar{k}_1$, and $\hat{E} < \bar{E}_1$.*

Proof of Proposition 4 *The first part of proposition follows directly from proof 1. The second part can be proven as follows. Define*

$$B(k) = \psi_x\eta/\gamma k^\alpha + \psi_0 - 2\psi_x k.$$

Since function $B(k)$ is positive for $k < \bar{k}_3$ and negative for $k > \bar{k}_3$, then it follows that $\hat{k} > \bar{k}_3$ if $B(\bar{k}) < 0$. Substituting the solution for $\bar{k} = (\eta/\gamma)^{\frac{1}{1-\alpha}}$ into $B(k)$ and assuming that $B(k) < 0$ yields

$$B(\hat{k}) = \psi_x\eta/\gamma(\eta/\gamma)^{\frac{\alpha}{1-\alpha}} + \psi_0 - 2\psi_x(\eta/\gamma)^{\frac{1}{1-\alpha}} < 0.$$

Simplifying gives the condition $\psi_0/\psi_x < (\eta/\gamma)^{\frac{1}{1-\alpha}} \equiv \hat{k}$. At this parameter configuration we know that Regime 3 applies, and consequently there is no contradiction. Thus $\hat{k} > \bar{k}_3$.

Assume now that $\hat{E} < \bar{E}_3$, thus

$$\bar{E}_3 = \alpha\psi_x\bar{k}_3^{1+\alpha} > \alpha\psi_0\bar{k}^\alpha.$$

This implies $\bar{k}_3 > (\psi_0/\psi_x)^{\frac{1}{1+\alpha}}(\eta/\gamma)^{\frac{\alpha}{1-\alpha^2}} \equiv \tilde{k}$. If $B(\tilde{k}) > 0$, then $\bar{k}_3 > \tilde{k}$ and consequently $\hat{E} < \bar{E}_3$. Evaluating $B(\tilde{k}) > 0$ gives the condition

$$(\psi_0/\psi_x)^{\frac{\alpha}{1+\alpha}}(\eta/\gamma)^{\frac{1}{1-\alpha^2}} + \psi_0/\psi_x - 2(\psi_0/\psi_x)^{\frac{1}{1+\alpha}}(\eta/\gamma)^{\frac{\alpha}{1-\alpha^2}} > 0.$$

This is equivalent to

$$\hat{k}^{\frac{1}{1+\alpha}} + (\psi_0/\psi_x)^{\frac{1}{1+\alpha}} - 2(\psi_0/\psi_x)^{\frac{1-\alpha}{1+\alpha}}\hat{k}^{\frac{\alpha}{1+\alpha}} > 0.$$

This condition holds if $1 - 2(\psi_0/\psi_x)^{\frac{1-\alpha}{1+\alpha}}\hat{k}^{\frac{\alpha-1}{1+\alpha}} > 0$, which can be re-written to $\hat{k} > 2^{\frac{1+\alpha}{1-\alpha}}\psi_0/\psi_x$. This holds always in Regime 3 since $2^{\frac{1+\alpha}{1-\alpha}} > 1$ and $\frac{\hat{k} > \psi_0}{\psi_x}$. Thus, we find that $\hat{E} < \bar{E}_3$. ■

Conclusively, by raising the importance of environmental quality for utility, investments in environmental culture lead to lower levels of consumption and higher environmental quality at steady state in comparison to the restricted model.

What we, thus, find is that wealth drives utility through higher consumption levels and the possibility to improve environmental quality through maintenance, but after a certain level of economic development there are other aspects that are raised into focus. In our case, the particular aspect is environmental culture. A society that is rich enough will invest in both culture and the environment, and thereby end up with a greater level of happiness than one that is unable to invest in environmental culture. This result suggests that, after a certain level of economic development, society may wish to undergo a social change that places cultural aspects, here environmental cultural aspects, at the forefront of decision-taking.

4.1 Robustness

In this section we study the robustness of the previous results. In particular, we first look at the role of the depreciation rate in environmental culture,

then we analyze the implication of allowing for a natural regeneration rate in environmental quality, and finally we look at the role of technical changes.

4.1.1 Role of depreciation rate in environmental culture

One assumption in our model is that environmental culture depreciates fully during the course of one generation. As a reminder, we assumed this since each new generation is born void of any environmental culture and needs to learn this culture through e.g. costly educational measures. However, cultural attitudes may also be learned through means other than costly education. An environmental culture may also simply be transferred from the parents to their children based on costless learning-by-doing, or simple adoption of behavioral habits. There is, for example, a growing literature on cultural learning, where the parents' preference traits affect the preferences of their children. In this regards, Graumann and Kruse (1990) find that attitudes are a social construct, implying that society plays a significant role in shaping preferences. Dalhouse and Frideres (1996) conclude that children tend to adopt the political position of their parents. Thus, if one assumes that one's political choices are driven by one's preferences, then this naturally leads one to accept that there is also an intergenerational transmission of environmental preferences. This intergenerational transmission of preferences is studied and analysed in a series of articles by Bisin and Verdier (2001), who emphasize the role of cultural dynamics and norms via social interactions across generations. Schumacher (2009a) builds upon their work and studies the effect of environmental quality on the intergenerational transmission of preferences. In an empirical study, Villacorta et al. (2003) conclude that children's environmental self-regulation is shaped by their parents' environmental self-regulation. Thus, there is ample evidence suggesting that an intergenerational transmission of preferences may take place.¹⁵

¹⁵The evidence does not necessarily imply that an intergenerational transmission of preferences is costless. It could simply be that the intergenerational transmission facili-

Furthermore, it may be that there are complements between existing levels of environmental culture and e.g. educational expenditure in environmental culture. In line with this literature, we assume that environmental culture may depreciate at a rate $\delta \in (0, 1)$, but also interacts with environmental educational investments, such that

$$X_{t+1} = \psi((1 - \delta)X_t, x_t).$$

Our assumption throughout the article was that $\delta = 1$ and $\psi_X = 0$. In this extension, we take it that $d\psi/dX > 0$, and there is an upper bound to which X_t tends for $x_t = (\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}})(1 - \alpha)^{\frac{1}{1-\alpha}}$ and $a_t = 0$.¹⁶ Finally, $X_0 > 0$.

Overall we observe that this does not modify the first-order condition, and neither the conditions in Proposition 1. The only qualitative change may then come from the dynamics of the model. Clearly, Regime 1 and 2 are unaltered, since they have no investments in environmental culture. Regime 3 and 4 will also see no qualitative changes to the dynamics themselves, only quantitative ones. However, it may be possible to find a non-monotonic convergence to the steady state in Regime 3. We leave this point for future research. Conclusively, there are no important differences to the existence of the steady states if one allows for less than full depreciation as well as feedbacks from the stock of environmental culture to investments in environmental culture.

4.1.2 Implication of no regeneration rate

Some might view the assumption of no regeneration rate in environmental quality as being too restrictive. We now relax this assumption and assume

tates the shaping of an environmental culture, e.g. through a higher marginal effect of investments in environmental culture.

¹⁶In other words, we define this upper bound for the maximum sustainable amount that an agent may invest in x_t . This is given by $k = (1 - \alpha)k^\alpha - x$, solving for x , maximizing this subject to k and substituting the optimal solution for k back into the solution for x .

the law of motion as introduced in Jouvet et al. (2005). This is given by

$$E_{t+1} = m\tilde{E} + (1 - m)E_t - \beta c_t + \gamma a_t,$$

where $m \in (0, 1)$ is the rate of convergence to the natural state $\tilde{E} > 0$. Clearly, for $m = 0$ we recover the functional form suggested in Assumption 4, while in the case of $m = 1$ there would be full convergence back to the natural level during the course of each generation. Our focus is on whether there are qualitative differences if one assumes $m \in (0, 1)$ in comparison to the case of $m = 0$ assumed above.

Based on the conditions identified in Proposition 1, there are only quantitative differences arising in the position of the four regions, since now $g(E_t, c_t) = m\tilde{E} + (1 - m)E_t - \beta c_t$ instead of $g(E_t, c_t) = E_t - \beta c_t$. Regime 1 and 2 will stay qualitatively the same, and so will Regime 3. The only qualitative difference is in Regime 4. A sufficiently high regeneration rate will lead to the two dynamic equations

$$E_{t+1} = m\tilde{E} + (1 - m)E_t - \alpha\beta k_t^\alpha, \quad (37)$$

$$k_{t+1} = \frac{\psi_x(1 - \alpha)k_t^\alpha + \psi_0}{2\psi_x}. \quad (38)$$

As a result, the unique steady state is defined by

$$E = \tilde{E} - \frac{\alpha\beta}{m}k^\alpha, \quad (39)$$

$$2\psi_x k = \psi_x(1 - \alpha)k^\alpha + \psi_0. \quad (40)$$

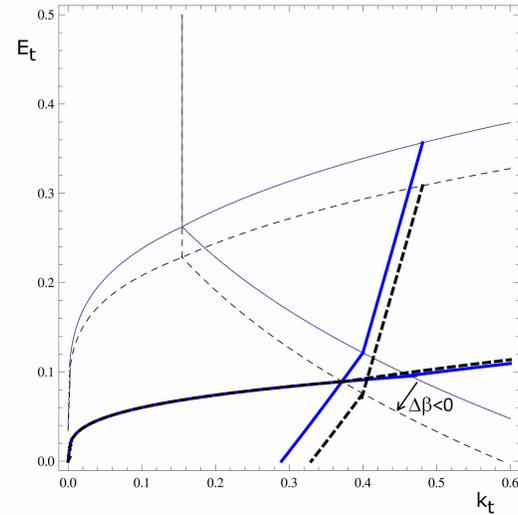
Linearizing around the unique steady state shows that the steady state in this case is asymptotically stable. However, this steady state exists only if $\tilde{E} > \frac{\alpha\beta}{m}k^\alpha$. Thus, for a low natural level of the environment, or a sufficiently slow regeneration rate, we find that our previous results continue to hold qualitatively even in the case of a more general law of motion for environmental quality.

4.2 Improvements in technology

In this section we investigate changes in β , γ and ψ_x . The analytical derivations for these comparative statics are available in Appendix 3. Figures 3, 4 and 5 give the results graphically. The dashed lines are the new curves that arise from a change in the parameter in question.

If β decreases, then this implies a more environmentally-friendly technology that leads to lower environmental damage from consumption. An improvement in the technology that leads to fewer environmental spillovers from consumption is shown in Figure 3. A decrease in β basically shifts all region curves down. The steady state curves $E = z_2(k)$ and $E = w_2(k)$ shift down, $E = z_1(k)$ stays constant while the slope of $E = w_1(k)$ increases. This leads to an increase in environmental quality and capital stock at steady state since the more efficient technology makes more wealth available that may be directed towards environmental culture or environmental quality. Both Region 3 and 4 expand in size, implying that an interior solution to environmental culture is now more likely. In our example in Figure 3, society was initially stuck in a low environmental culture equilibrium. The improvement in environmentally-friendly technology leads to an interior solution in environmental culture, and consequently to a steady state in Region 3. Thus, even small improvements in the environmental-friendliness of production technologies may lead to enough wealth being freed up for investments in environmental culture that then induces positive feedback loops over time, where more environmental culture leads to more environmental quality which in turn induces another generation to invest more in environmental culture. One remark may be in order here. If the production technology becomes more environmentally-friendly at time t , then this will have no impact on the decisions of the young generation at time t , since β affects a generation's choices only through the level of consumption. Consequently, a change in production technology at time t will only impact the decisions of the young generation at time $t + 1$. This, furthermore, implies that, in this model of

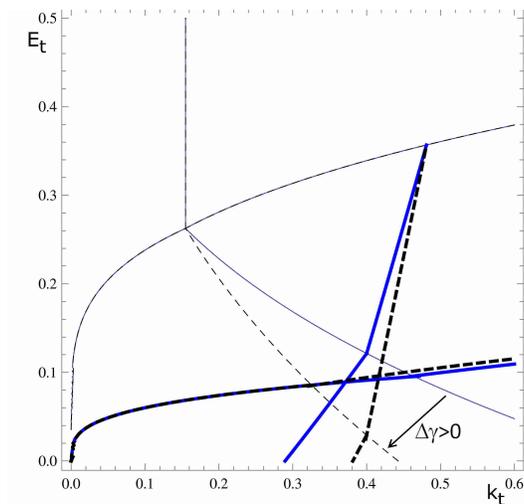
Figure 3: Changes in β



intergenerational spillovers, there are no incentives for the young generation at time t to make the production technology more environmentally-friendly. Only a policy maker who, at minimum, maximizes over two periods, will have an incentive to finance improvements in the consumption spillover on environmental quality. This result indicates that e.g. costly R&D expenditure may not be undertaken in case society's benefit is only in the distant future, and therefore society may never be able to free enough wealth to get out of the low environmental culture and environmental quality equilibrium.

An increase in γ implies a more efficient abatement technology. This is depicted in Figure 4. The only region curve that shifts is $g_t = g_L$ which separates Region 1 and 3. Just like in the case for decreases in β , an increase in γ shifts the steady state curves $E = z_2(k)$ and $E = w_2(k)$ down, $E = z_1(k)$ stays constant while the slope of $E = w_1(k)$ increases. Thus, an improvement in the maintenance technology makes a given amount of abatement more efficient in improving environmental quality. In Figure 4, the improvement in abatement technology leads to an interior solution in environmental culture, and consequently to a steady state in Region 3. The fundamental difference

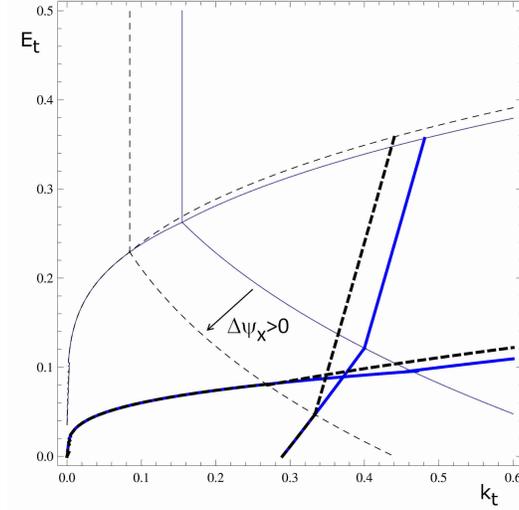
Figure 4: Changes in γ



between improvements in emissions per unit of consumption (i.e. reductions in β) and improvements in abatement technology (i.e. increases in γ) is that changes in β have an impact on the given level of environmental quality $g(E_t, c_t)$. This diminishes the need to improve environmental quality via maintenance, thus shrinking region 1. The increase in given environmental quality makes investments in environmental culture more profitable, thus expanding Region 3 and 4. Improvements in the effectiveness of maintenance only impact the trade-off between investments in environmental culture and abatement. Consequently, a better maintenance technology allows to direct more wealth towards environmental culture. This results in a shrinking of Region 1 and an expansion of Region 3. In contrast to changes in β , costly R&D expenditure that affects γ may be worthwhile to undertake for a young generation since changes in the effectiveness of abatement expenditure today impact environmental quality when old and thus affect the young generation's choices already today.

Finally, an improvement in ψ_x implies a more efficient learning process in environmental culture. Consequently, the regions 3 and 4 in which invest-

Figure 5: Changes in ψ_x



ments in environmental culture are worthwhile to undertake expand. While the steady state curves $E = z_1(k)$ and $E = z_2(k)$ remain unchanged, the slopes of both $E = w_1(k)$ and $E = w_2(k)$ become steeper. Thus, at steady state the agent gives up some consumption in order to achieve a higher level of environmental quality at steady state. Thus, the agent raises both investments in environmental culture and in maintenance.

We can observe that in all three cases (changes in β , γ or ψ_x) it is possible that technological improvements lead to shift from an equilibrium without environmental culture, low environmental quality and low wealth, to one with environmental culture, higher environmental quality and higher wealth. Thus, the technological improvements studied here have the potential to lead to an Environmental Kuznets Curve (EKC). An EKC arises if the relationship between economic development and environmental quality is inversely u-shaped. For example, let us assume that environmental quality was high but wealth was low. Let us furthermore take it that the parameter conditions lead society to eventually pick up the steady state in Regime 1. In this case, society will find itself in Regime 2, and it will increase wealth at the

expense of environmental quality. When environmental quality is run down to a sufficiently low level, then society will start to invest in maintenance, and we observe a convergence to a medium level of wealth but a low level of environmental quality. Assume now that there is a sufficiently large technological advance in how consumption leads to pollution, or in the effectiveness of abatement, or in education. In this case, society may start to invest in environmental culture, leading to higher levels of environmental quality and a further economic development. Thus, an EKC may arise in this model, but it requires sufficiently large technological improvements that make society pick up the steady state in Regime 3 rather than the one in Regime 1.

5 Conclusion

In this article we study the role of an endogenous environmental culture in an overlapping generation model with Leontieff preferences. Both an endogenous environmental culture as well as Leontieff preferences have seen little emphasis in the environmental economics literature. However, culture has been emphasized time and again as one of the important pillars of sustainable development, while the Leontieff preferences are believed to be an empirical regularity.

Our main findings are that for low wealth levels, society is unable to free resources for environmental culture. In this case, society will only invest in environmental maintenance if environmental quality is sufficiently low. Once society has reached a certain level of economic development, then it may optimally invest a part of its wealth in developing an environmental culture. When environmental quality and wealth are both sufficiently high, then society may find it optimal to temporarily over-invest in environmental culture. This is optimal until environmental quality is decreased to a level from which onwards it is worthwhile for society to also invest in environmental quality. In other words, if there is no urgent need for society to improve environmen-

tal quality, then society will either invest in an environmental culture if it can afford to do so, or not invest in case it is too poor. Technological improvements in emission or abatement efficiency both raise steady state wealth and environmental quality.

Our model suggests that, by raising the importance of environmental quality for utility, environmental culture leads to lower levels of consumption and higher investments in maintenance in contrast to a restricted model where no investments in environmental culture are possible. It, furthermore, leads to societies with lower steady state levels of wealth but higher steady state levels of environmental quality. Furthermore, since investments in environmental culture raise the importance of environmental quality for utility, it becomes also more worthwhile for society to invest in maintenance due to the positive feedback between culture and the environment. As a result, environmental culture has not only a positive impact on environmental quality through lower levels of consumption, but in addition it improves the environment through maintenance expenditure for wealth-environment combinations at which, without environmental culture, no maintenance would be undertaken.

This analysis suggest that wealth drives happiness (utility) through higher consumption levels and the possibility to improve environmental quality through maintenance, but after a certain level of economic development there are other aspects that are raised into focus. In our case, the particular aspect we looked at is environmental culture. A society that is rich enough will invest in both culture and the environment, and thereby end up with a greater level of happiness than one that is unable to invest in environmental culture. This result suggests that, after a certain level of economic development, society may wish to undergo a social change that places cultural aspects, here environmental cultural aspects, at the forefront of decision-taking.

In terms of future research, one suggestion would be to add to this analysis the last type of capital that we neglected here, namely human capital

(Becker 2009). One could then study the model within an endogenous growth setting and analyze further the interaction between the different types of capital along the path of economic development. Another extension could place this analysis within the literature of the Environmental Kuznets Curve (EKC). The idea would be to see under what circumstances environmental culture may drive a u-shaped relationship between the stages of economic development and environmental quality. For example, within this model an EKC will arise if society starts with a high environmental quality and low wealth combination, which leads to economic development at the expense of environmental quality and convergence to a steady state without environmental culture but some maintenance expenditure. If society then receives a windfall increase in capital (e.g. international capital inflows, remittances), or sufficiently large technological improvement in either the effectiveness of abatement or in the consumption-to-pollution spillover, then society may invest environmental culture, leading to higher levels of environmental quality and a further economic development. Thus, an EKC may arise in this model, but it requires technological improvements or windfall increases in wealth. As a result, what would be interesting would be to study a framework with endogenous technological changes or international capital markets.

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Appendix 1

Proof of Proposition 1 *Given the maximization problem as defined by eq. (3) to (6), we denote an optimal, regime-dependent solution by x_{it}^* , a_{it}^* and s_{it}^* , for $i = 1, \dots, 4$. Substituting the constraints into $E_{t+1} = \psi(x_t)c_{t+1}$ gives the reduced-form utility function, which we denote by*

$$W(x_t) = \frac{\psi(x_t)}{\gamma + \psi(x_t)R_{t+1}} \left(\gamma(w_t - x_t) + g(E_t, c_t) \right), \quad (41)$$

where the choice variable is x_t , which implicitly determines a_{it}^* and s_{it}^* . Maximizing function $W(x_t)$ subject to x_t gives rise to the first-order condition in eq. (12). The second-order condition is given by equation (13). It is negative and thus assures a maximum.

We define $A(x) \equiv \psi'(x) \left(\gamma(w_t - x) + g(E_t, c_t) \right)$ and $B(x) \equiv \psi(x) (\gamma + \psi(x)R_{t+1})$. These have the properties that $A(0) = \psi'(0) (\gamma w_t + g(E_t, c_t)) > 0$, while $A(w_t) = \psi'(w_t) g(E_t, c_t) > 0$, with $A'(x_t) = \psi''(x_t) (\gamma(w_t - x_t) + g(E_t, c_t)) - \psi'(x_t) \gamma < 0$. In contrast, $B(0) = \psi(0) (\gamma + \psi(0)R_{t+1}) > 0$, $B(w_t) = \psi(w_t) (\gamma + \psi(w_t)) > 0$ with $B(w_t) > B(0)$, and $B'(x_t) = \psi'(x_t) (\gamma + 2\psi(x_t)) > 0$.

Thus, there exists a unique solution to eq. (12) if $A(0) > B(0)$ and $B(w_t) > A(w_t)$. Condition $A(0) > B(0)$ can be re-written as

$$g(E_t, c_t) > \frac{\psi(0)}{\psi'(0)} (\gamma + \psi(0)R_{t+1}) - \gamma w_t \equiv g_L,$$

while $A(w_t) < B(w_t)$ leads to

$$g(E_t, c_t) < \frac{\psi(w_t)}{\psi'(w_t)} (\gamma + \psi(w_t)R_{t+1}) \equiv g_H.$$

There exists a $g(E_t, c_t)$ such that $g(E_t, c_t) \in (g_L, g_H)$ since $g_L < g_H$ holds always if Assumption 1 applies.

Solving eq. (12) for x_t gives the solution $x_t = x_{3t}^*$. Substituting this x_{3t}^* into eq. (8) then leads to the following conditions. If $\Gamma(x_{3t}^*) > g(E_t, c_t)$ then $a_{3t}^* = \frac{\Gamma(x_{3t}^*) - g(E_t, c_t)}{\psi(x_{3t}^*)R_{t+1} + \gamma}$, with $s_{3t}^* = w_t - a_{3t}^* - x_{3t}^*$ (**Regime 3**).

If $g(E_t, c_t) \in (g_L, g_H)$ and $\Gamma(x_{3t}^*) \leq g(E_t, c_t)$ then $a_{4t}^* = 0$ and $s_{4t}^* = w - x_{4t}^*$. Since $\Gamma(x_{3t}^*)$ is also implicitly a function of $g(E_t, c_t)$, then we need to know the position of $\Gamma(x_{3t}^*)$ relative to $\Gamma(x_t^m)$ and g_H when $\Gamma(x_{3t}^*) \leq g(E_t, c_t)$. This we do as follows. We take the first-order condition eq (12) and set $g(E_t, c_t) = \Gamma(x_{3t}^*)$. This gives us

$$\psi'(x_{3t}^*) \left(\gamma(w_t - x_{3t}^*) + \psi(x_{3t}^*)R_{t+1}(w_t - x_{3t}^*) \right) = \psi(x_{3t}^*)(\gamma + \psi(x_{3t}^*)R_{t+1}).$$

Simplifying leads to $\psi'(x_{3t}^*)(w_t - x_{3t}^*) = \psi(x_{3t}^*)$. This only holds if $x_{3t}^* = x_t^m$. There exist two sub-cases which are based on Lemma 1. An interior solution to $\psi'(x_{3t}^*)(w_t - x_{3t}^*) = \psi(x_{3t}^*)$ requires $w_t > \psi(0)/\psi'(0)$, and we denote the optimal solution in this case as $x_{4t}^* = x_t^m$. Thus, for $\Gamma(x_{3t}^*) \leq g(E_t, c_t)$ and $w_t > \psi(0)/\psi'(0)$ we have that $x_{4t}^* = x_t^m$ (**Regime 4**). If $w_t \leq \psi(0)/\psi'(0)$, then there exists no interior solution to $\psi'(x_{3t}^*)(w_t - x_{3t}^*) = \psi(x_{3t}^*)$ and the optimal x_t^* will be equal to zero. We denote this as x_{2t}^* (**Regime 2**).

For $A(0) \leq B(0)$, then this implies $g(E_t, c_t) \leq g_L$. Hence, $W'(x_t) < 0$, $\forall x_t \leq w_t$, and thus $x_t^* = 0$ and $a_t^* \geq 0$, where a_t^* solves eq. (8) with $x_t^* = 0$ and $s_t^* = w - a_t^*$. If $g(E_t, c_t) < \psi(0)R_{t+1}w_t$ then from eq. (8) at $x_{1t}^* = 0$ we obtain $a_{1t}^* = \frac{\psi(0)R_{t+1}w_t - g(E_t, c_t)}{\gamma + \psi(0)R_{t+1}}$ (**Regime 1**), while $a_{2t}^* = 0$ if $g(E_t, c_t) \geq \psi(0)R_{t+1}w_t$ (**Regime 2**). ■

Proof of Proposition 2 The steady state of this system can easily be calculated by solving the dynamic system (14) and (15) for their fixed points k and E . We denote the steady state values of k_t and E_t in regime 1 by \bar{k}_1 and

\bar{E}_1 . Solving for these gives the two steady states

$$\bar{k}_1 = \left(\frac{\eta}{\gamma}\right)^{\frac{1}{1-\alpha}}, \quad (42)$$

$$\bar{E}_1 = \alpha\psi_0 \left(\frac{\eta}{\gamma}\right)^{\frac{\alpha}{1-\alpha}}. \quad (43)$$

For stability we linearize system (14) and (15) around the unique steady state and obtain the system

$$\begin{bmatrix} E_{t+1} \\ k_{t+1} \end{bmatrix} = \begin{bmatrix} \bar{E}_1 \\ \bar{k}_1 \end{bmatrix} + \begin{bmatrix} \frac{\alpha^2\psi_0}{\eta+\alpha^2\psi_0} & \frac{\alpha^3\gamma\psi_0}{\eta+\alpha^2\psi_0} \\ \frac{\eta}{\gamma(\eta+\alpha^2\psi_0)} & \frac{\alpha\eta}{\eta+\alpha^2\psi_0} \end{bmatrix} \begin{bmatrix} E_t - \bar{E}_1 \\ k_t - \bar{k}_1 \end{bmatrix},$$

From this we obtain the characteristic polynomial

$$\lambda_1^2 - \frac{\alpha(\eta + \alpha\psi_0)}{\eta + \alpha^2\psi_0}\lambda_1 = 0 \quad (44)$$

This gives rise to the eigenvalues $\lambda_{1a} = 0$ and $\lambda_{1b} = \frac{\alpha(\eta+\alpha\psi_0)}{\eta+\alpha^2\psi_0}$. It is then straight-forward to see that $\lambda_{1b} \in (0, 1)$, which implies a monotonic convergence to the steady state. ■

Proof of Proposition 3 First we show that $g_L < g_H$. Combining and replacing the explicit functional forms gives, after simplification, $-\alpha(2\psi_0k^{\alpha-1} + \psi_x(1-\alpha)k^{2\alpha-1}) < 2\gamma$. This thus holds at any interior k .

We then look at equation (32). If $\left(\psi_0 + \psi_x \left[\frac{\eta}{\gamma}k^\alpha - k\right]\right) \alpha k^{\alpha-1} \neq -\gamma$, then the equation describing the steady state is given by

$$\psi_x k = \psi_0 + \psi_x \left[\frac{\eta}{\gamma}k^\alpha - k\right]. \quad (45)$$

If instead equation (45) does not hold, then the steady state equation is defined by

$$\alpha \left(\psi_0 + \psi_x \left[\frac{\eta}{\gamma}k^\alpha - k\right]\right) = -\gamma k^{1-\alpha}. \quad (46)$$

A third possibility is that both equations (45) and (46) hold with equality. This is the easiest to dismiss since combining both equations leads to $\alpha\psi_x k^\alpha = -\gamma$ which is impossible. We now need to study whether the two potential steady states exist within the bounds given by Proposition 1. For equation (46) it is easy to show that $g > g_L$ never holds. Substituting equation (46) into the threshold condition $g_t > g_L$ gives, after some simplifications,

$$\eta k^\alpha - \gamma k > \frac{\psi_0}{\psi_x} (\gamma + \psi_0 \alpha k^{\alpha-1}).$$

The right-hand side is positive, while the left-hand side is negative at the steady state described by equation (46). Thus, the solution for k given by equation (46) will not define a steady state for case 3.

For equation (45) we can show that $g < g_H$ if $k > \psi_0/\psi_x$. Assuming $g < g_H$ requires

$$\left(\psi_0 + \psi_x \left[\frac{\eta}{\gamma} k^\alpha - k \right] \right) \alpha k^\alpha - \alpha \beta k^\alpha < \quad (47)$$

$$\frac{\psi_0 + \psi_x (1 - \alpha) k^\alpha}{\psi_x} (\gamma + (\psi_0 + \psi_x (1 - \alpha) k^\alpha) \alpha k^{\alpha-1}). \quad (48)$$

We then collect terms and slightly rewrite to get

$$(\psi_0 + \psi_x (1 - \alpha) k^\alpha) (\psi_x \alpha k^{\alpha-1} - \gamma k - (\psi_0 + \psi_x (1 - \alpha) k^\alpha) \alpha k^\alpha) < -\Psi,$$

where $\Psi \equiv -\psi_x^2 (\alpha \beta / \gamma k^\alpha - k) \alpha k^\alpha - \psi_x \alpha \beta k^\alpha < 0$. Dividing through by the positive term $(\psi_0 + \psi_x (1 - \alpha) k^\alpha)$ and substituting equation (45) for k^α , multiplying by $\eta (> 0)$ leads to

$$\psi_x \alpha \eta k^{\alpha+1} - \eta \gamma k - \psi_0 \alpha \eta k^\alpha - \alpha (1 - \alpha) \gamma (2\psi_x k - \psi_0) k^\alpha < \tilde{\Psi} k,$$

where $\tilde{\Psi} = \frac{\eta}{\psi_0 + \psi_x (1 - \alpha) k^\alpha} \Psi$. Collecting terms and simplifying gives

$$-\alpha \beta (\psi_x k - \psi_0) k^\alpha - \eta k - \alpha (1 - \alpha) \gamma \psi_x k^{\alpha+1} < \tilde{\Psi} k.$$

A sufficient condition for this inequality to hold is $k > \psi_0/\psi_x$. Finally, we derive the condition under which equation (45) implies $g > g_L$. Assuming $g > g_L$ and substituting equation (45) gives

$$\psi_x \alpha k^{\alpha+1} > \frac{\psi_0}{\psi_x} (\gamma + \psi_0 \alpha k^{\alpha-1}) - \eta k^\alpha.$$

Slightly rewriting gives us

$$\eta + \psi_x \alpha k > \frac{\psi_0}{\psi_x} (\gamma k^{-\alpha} + \psi_0 \alpha k^{-1}).$$

Substituting equation (45) again gives

$$\eta \psi_x k \left(2 \frac{\psi_x k - \psi_0}{2\psi_x k - \psi_0} \right) + \psi_x^2 \alpha k^2 > \psi_0^2 \alpha.$$

Clearly, if $k > \psi_0/\psi_x$, then this inequality is satisfied. If $k \in \left[\frac{\psi_0}{2\psi_x}, \frac{\psi_0}{\psi_x} \right)$, then $g < g_L$. Finally, define $\Omega(k) \equiv \eta k \left(2 \frac{\psi_x k - \psi_0}{2\psi_x k - \psi_0} \right) + \psi_x^2 \alpha k^2$. Then it is easy to show that $\Omega'(k) > 0$ if $k < \frac{\psi_0}{2\psi_x}$. Thus, we conclude that $g < g_L, \forall k < \psi_0/\psi_x$.

For stability, we proceed as follows. We linearize system (28) and (29) around the unique steady state to obtain the system

$$\begin{bmatrix} E_{t+1} \\ k_{t+1} \end{bmatrix} = \begin{bmatrix} E \\ k \end{bmatrix} + DH(E, k) \begin{bmatrix} E_t - E \\ k_t - k \end{bmatrix},$$

where matrix $DH(E, k)$ takes the form

$$DH(E, k) = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}.$$

The individual elements of matrix $DH(E, k)$ are given by

$$H_{11} = \frac{k^\alpha \alpha (1 + \alpha) \psi_x}{2\gamma + k^\alpha \alpha (1 + \alpha) \psi_x},$$

$$H_{12} = \frac{k^{-1+2\alpha}\alpha^2(1+\alpha)\gamma\psi_x(\eta - \alpha(\psi_0 - 2k\psi_x))}{(\gamma + k^\alpha\alpha\psi_x)(2\gamma + k^\alpha\alpha(1+\alpha)\psi_x)},$$

$$H_{21} = \frac{1}{2\gamma + k^\alpha\alpha(1+\alpha)\psi_x},$$

$$H_{22} = \frac{\alpha\eta}{k(2k^{-\alpha}\gamma + \alpha(1+\alpha)\psi_x)}.$$

From this we obtain the characteristic polynomial

$$\lambda^2 - \frac{\alpha(\eta + k\psi_x + \alpha k\psi_x)}{k(2k^{-\alpha}\gamma + \alpha(1+\alpha)\psi_x)}\lambda = 0 \quad (49)$$

This gives rise to the eigenvalues $\lambda_1 = 0$ and $\lambda_2 = \frac{\alpha(\eta + k\psi_x + \alpha k\psi_x)}{k(\alpha(1+\alpha)\psi_x + \frac{2\eta\psi_x}{2k\psi_x - \psi_0})}$. Then $\lambda_2 < 1$ implies $\psi_0 - 2k\psi_x < 2\eta$, which holds under Assumption 5 and iff $k > \psi_0/\psi_x$; and $\lambda_2 > -1$ implies $\alpha\eta > -2k\psi_x \left(\alpha(1-\alpha) + \frac{\eta}{2k\psi_x - \psi_0} \right)$. Again, this holds if $k > \psi_0/\psi_x$ and under Assumption 5. Finally, it is easy to see that $\lambda_2 > 0$. Thus, there are no cycles and convergence to the steady state is monotonic. ■

Appendix 2

DERIVATION OF THE FIGURES IN SECTION 4

Here we derive the shape of the four different regions that depend on the analytical thresholds. As a reminder, those thresholds are

$$\begin{aligned} g_L &= \frac{\psi_0}{\psi_x}(\gamma + \psi_0\alpha k_{t+1}^{\alpha-1}) - \gamma(1 - \alpha)k_t^\alpha, \\ g_H &= \frac{\psi_0 + \psi_x(1 - \alpha)k_t^\alpha}{\psi_x}(\gamma + (\psi_0 + \psi_x(1 - \alpha)k_t^\alpha)\alpha k_{t+1}^{\alpha-1}), \\ \Gamma(x_t^m) &= \alpha k_{t+1}^{\alpha-1} \frac{(\psi_x(1 - \alpha)k_t^\alpha + \psi_0)^2}{4\psi_x}. \end{aligned}$$

$\Gamma(x_t^m)$ is only defined for $(1 - \alpha)k_t^\alpha \geq \frac{\psi_0}{\psi_x}$.

The conditions that are separating the four regions are then as follows.¹⁷ Separating Regime 1 and Regime 3 is $g_t = g_L$, which becomes

$$E_t = \frac{\psi_0}{\psi_x}(\gamma + \psi_0\alpha k_{t+1}^{\alpha-1}) - \eta k_t^\alpha. \quad (50)$$

In the limit to this point we know that equation (14) holds

$$k_{t+1} = \frac{E_t + \eta k_t^\alpha}{\gamma + \psi_0\alpha k_{t+1}^{\alpha-1}}.$$

Substituting the equation above into (50) and solving for k_{t+1} gives $k_{t+1} = \psi_0/\psi_x$. Substituting this solution into the equation above yields

$$E_t = \frac{\psi_0}{\psi_x}(\gamma + \psi_0\alpha(\psi_0/\psi_x)^{\alpha-1}) - \eta k_t^\alpha.$$

By the constraints for Regime 3, this equation starts from $w_t = \psi_0/\psi_x$, which

¹⁷Here we remind the reader that the subsequent solutions *only* apply to separate the regimes in question, and do not hold for regions in which other regimes may apply.

implies

$$E_t = \alpha\psi_0(\psi_0/\psi_x)^\alpha + \frac{\alpha\beta}{1-\alpha}\psi_0/\psi_x,$$

while its slope is given by

$$\frac{dE_t}{dk_t} = -\alpha\eta k_t^{\alpha-1} < 0,$$

and its second derivative is

$$\frac{d^1 E_t}{dk_t^1} = \alpha(1-\alpha)\eta k_t^{\alpha-2} > 0,$$

It cuts $E_t = 0$ at

$$k_t = \left[\frac{\psi_0}{\eta\psi_x} (\gamma + \psi_0\alpha(\psi_0/\psi_x)^{\alpha-1}) \right]^{\frac{1}{\alpha}}.$$

Regime 3 and 4 are separated by the condition $g_t = \Gamma(x_t^m)$. Substituting the explicit solutions and the dynamic equation for capital in Regime 4, we can re-write this as

$$E_t = \alpha\beta k_t^\alpha + \frac{\alpha}{2^{\alpha+1}\psi_x^\alpha} \left[\psi_0 + \psi_x(1-\alpha)k_t^\alpha \right]^{\alpha+1}.$$

By the constraints for Regime 3 and 4, this equation starts at $w_t = \psi_0/\psi_x$, giving

$$E_t = \alpha\psi_0(\psi_0/\psi_x)^\alpha + \frac{\alpha\beta}{1-\alpha}\psi_0/\psi_x,$$

which is equivalent to the starting point of $g_t = g_L$ that separates Regime 1 and 3. Its slope is given by

$$\frac{dE_t}{dk_t} = \alpha^2\beta k_t^{\alpha-1} + \frac{\alpha^2(1-\alpha^2)}{2^{\alpha+1}\psi_x^{\alpha-1}} \left[\psi_0 + \psi_x(1-\alpha)k_t^\alpha \right]^\alpha k_t^{\alpha-1} > 0.$$

The condition that separates Regime 1 and 2 is $g_t = \psi_0 R_{t+1} w_t$, which, when

combined with the capital equation for Regime 2, yields

$$E_t = \alpha\beta k_t^\alpha + \alpha\psi_0(1 - \alpha)^\alpha k_t^{\alpha^2}.$$

For $k_t = 0$ we obtain $E_t = 0$, while the derivative is

$$\frac{dE_t}{dk_t} = \alpha^2\beta k_t^{\alpha-1} + \alpha^3\psi_0(1 - \alpha)^\alpha k_t^{\alpha^2-1} > 0,$$

and the second derivative is

$$\frac{d^2E_t}{dk_t^2} = -(1 - \alpha)\alpha^2\beta k_t^{\alpha-2} - \alpha^3\psi_0(1 - \alpha)^\alpha(1 - \alpha^2)k_t^{\alpha^2-2} < 0.$$

By the constraints for Regime 1 and 2, this function reaches its maximum at $w_t = \psi_0/\psi_x$, which gives

$$E_t = \alpha\psi_0(\psi_0/\psi_x)^\alpha + \frac{\alpha\beta}{1 - \alpha}\psi_0/\psi_x,$$

which again is the same point as for the other constraints. Finally, Regime 2 and 4 are separated by the vertical line $w_t = \psi_0/\psi_x$.

We can derive the steady state curves as follows. For Regime 1 they will be given by the time-constant versions of the equations (14) and (15), which, after solving for E , are respectively given by

$$E = \psi_0\alpha\eta k^\alpha \equiv z_1(k), \tag{51}$$

$$E = \gamma k + (\psi_0\alpha - \eta)k^\alpha \equiv z_2(k). \tag{52}$$

Then the shape of $z_1(k)$ is $z_1(0) = 0$, $z_1'(k) > 0$ and $z_1''(k) < 0$. The shape of $z_2(k)$ is $z_2(0) = 0$, $z_2'(0) < (>)0$ if $\alpha\psi_0 < (>)\eta$. If $\alpha\psi_0 < \eta$ then $z_2'(k) > 0$ for $k > \left[\frac{\alpha(\eta - \alpha\psi_0)}{\gamma}\right]^{\frac{1}{1-\alpha}}$.

The steady state curves of Regime 3 are the time-constant versions of

equations (28) and (29), which are respectively given by

$$E = (\psi_0 + \psi_x \rho) \alpha k^\alpha, \quad (53)$$

$$\psi_0 \alpha k^\alpha + \gamma k = E + \eta k^\alpha - (\psi_x \alpha k^\alpha + \gamma) \rho, \quad (54)$$

with

$$\rho = \frac{1}{\alpha \psi_x k^{\alpha-1}} \left[-(\gamma + k^{\alpha-1} \alpha \psi_0) + \sqrt{\gamma^2 + \alpha k^{\alpha-1} (\gamma \psi_0 + E \psi_x + \psi_x \eta k^\alpha)} \right].$$

Substituting and solving for E yields the following two steady state equations

$$E = \gamma \left(2k - \frac{\psi_0}{\psi_x} \right) + (\alpha k \psi_x - \eta) k^\alpha \equiv w_1(k), \quad (55)$$

$$E = \frac{1}{2} \left[-2\gamma k + \alpha k^{\alpha+1} \psi_x + \sqrt{k(k(\alpha \psi_x k^\alpha - 2\gamma)^2 + 4\alpha k^\alpha (\gamma \psi_0 + \eta \psi_x k^\alpha))} \right] \equiv w_2(k). \quad (56)$$

We can then derive the shape of these steady state curves as follows. We obtain $w_2(0) = 0$, with

$$w_2'(k) = \frac{1}{2} \left(-2\gamma + \alpha(1 + \alpha) k^\alpha \psi_x + \frac{4\gamma^2 k + \alpha k^{2\alpha} \psi_x ((2 + 4\alpha)\eta + \alpha(1 + \alpha)k\psi_x) + 2\alpha\gamma k^\alpha ((1 + \alpha)\psi_0 - (2 + \alpha)k\psi_x)}{\sqrt{k(k(-2\gamma + \alpha k^\alpha \psi_x)^2 + 4\alpha k^\alpha (\gamma \psi_0 + \eta k^\alpha \psi_x))}} \right),$$

and

$$w_2''(k) = \frac{1}{2} \left(\alpha^2(1+\alpha)k^{-1+\alpha}\psi_x \right. \\ \left. - \frac{(4\gamma^2k + \alpha k^{2\alpha}\psi_x((2+4\alpha)\eta + \alpha(1+\alpha)k\psi_x) + 2\alpha\gamma k^\alpha((1+\alpha)\psi_0 - (2+\alpha)k\psi_x))^2}{\left(k \left(k(-2\gamma + \alpha k^\alpha\psi_x)^2 + 4\alpha k^\alpha(\gamma\psi_0 + \eta k^\alpha\psi_x)\right)\right)^{3/2}} \right. \\ \left. + \frac{4\gamma^2k + \alpha^2(1+2\alpha)k^{2\alpha}\psi_x(4\eta + (1+\alpha)k\psi_x) + 2\alpha(1+\alpha)\gamma k^\alpha(\alpha\psi_0 - (2+\alpha)k\psi_x)}{k\sqrt{k \left(k(-2\gamma + \alpha k^\alpha\psi_x)^2 + 4\alpha k^\alpha(\gamma\psi_0 + \eta k^\alpha\psi_x)\right)}} \right).$$

Calculations for $w_1(k)$ give

$$w_1'(k) = 2\gamma - \alpha\eta k^{\alpha-1} + (1+\alpha)\alpha\psi_x k^\alpha,$$

which is negative for small capital ($w_1(0) = -\gamma\psi_0/\psi_x < 0$) and positive for sufficiently large capital,

$$w_1''(k) = (1-\alpha)\alpha\eta k^{\alpha-2} + \alpha^2(1+\alpha)\psi_x k^{\alpha-1} > 0.$$

Thus, $w_1(k)$ is a convex function, that starts from zero, turns negative and then positive.

We now prove that $w_1(k)$ is the continuation of $z_1(k)$ while $w_2(k)$ starts where $z_2(k)$ ends. We re-write the condition $g = g_L$ to

$$\alpha k^{\alpha-1} = \frac{1}{\psi_0^2} \left[\psi_x(E + \eta k^\alpha) - \gamma\psi_0 \right],$$

and substitute this into the time-constant solution for ρ . This gives

$$\rho = \frac{1}{\alpha\psi_x k^{\alpha-1}} \left[-\gamma - \alpha\psi_0^{\alpha-1} + \frac{\psi_x}{\psi_0}(E + \eta k^\alpha) \right].$$

Re-writing $g = g_L$ again gives us $E - \eta k^\alpha = \frac{\psi_0}{\psi_x}(\gamma + \psi_0\alpha k^{\alpha-1})$, and substituting this one into the equation for ρ above yields $\rho = 0$. Thus, at $g = g_L$ we have

$\rho = 0$. Consequently, we know that at $g = g_L$ we have

$$E = \psi_0 \alpha k^\alpha, \quad (57)$$

$$E = \psi_0 \alpha k^\alpha + \gamma k - \eta k^\alpha. \quad (58)$$

The first equation is equivalent to $z_1(k)$, while the second one is the same as $z_2(k)$. Thus, both sets of equations meet at $g = g_L$.

Appendix 3

DERIVATION OF RESULTS IN SECTION 4.2

Here we derive the comparative statics of the conditions that separate the four regimes, where we look at changes in β , γ and ψ_x . As a reminder, $\eta = (1 - \alpha)\gamma - \alpha\beta$.

We obtained that $g_t = g_L$ implies

$$E_t = \frac{\psi_0}{\psi_x}(\gamma + \psi_0\alpha(\psi_0/\psi_x)^{\alpha-1}) - \eta k_t^\alpha.$$

Thus, comparative statics lead to

$$\frac{dE_t}{d\beta} = \alpha k_t^\alpha > 0, \quad (59)$$

$$\frac{dE_t}{d\gamma} = \frac{\psi_0}{\psi_x} - (1 - \alpha)k_t^\alpha < 0, \quad (60)$$

$$\frac{dE_t}{d\psi_x} = -\frac{\gamma\psi_0}{\psi_x} - \alpha^2 \left(\frac{\psi_0}{\psi_x}\right)^{\alpha+1} < 0. \quad (61)$$

Then, $g_t = \Gamma(x_t^m)$ leads to

$$E_t = \alpha\beta k_t^\alpha + \frac{\alpha}{2^{\alpha+1}\psi_x^\alpha} \left[\psi_0 + \psi_x(1 - \alpha)k_t^\alpha \right]^{\alpha+1}.$$

In this case, comparative statics imply

$$\frac{dE_t}{d\beta} = \alpha k_t^\alpha > 0, \quad (62)$$

$$\frac{dE_t}{d\gamma} = 0, \quad (63)$$

$$\frac{dE_t}{d\psi_x} = \frac{\alpha\psi_x^{\alpha+1}(\psi_0 + \psi_x(1 - \alpha)k_t^\alpha)(\psi_x(1 - \alpha)k_t^\alpha - \alpha\psi_0)}{2^{\alpha+1}} > 0. \quad (64)$$

Also, $g_t = \psi_0 R_{t+1} w_t$ yields

$$E_t = \alpha\beta k_t^\alpha + \alpha\psi_0(1 - \alpha)^\alpha k_t^{\alpha^2}.$$

The comparative statics in this case give

$$\frac{dE_t}{d\beta} = \alpha k_t^\alpha > 0, \quad (65)$$

$$\frac{dE_t}{d\gamma} = 0, \quad (66)$$

$$\frac{dE_t}{d\psi_x} = 0. \quad (67)$$

Finally, $w_t = \psi_0/\psi_x$ gives

$$(1 - \alpha)k_t^\alpha = \psi_0/\psi_x.$$

Consequently, there is no change in E_t when β or γ change, but for changes in ψ_x this vertical threshold shifts to the left as given by $\frac{dk_t}{d\psi_x} = -\frac{1}{\alpha} \left(\frac{\psi_0}{1-\alpha} \right)^{1/\alpha} \psi_x^{-\frac{1-\alpha}{\alpha}} < 0$.