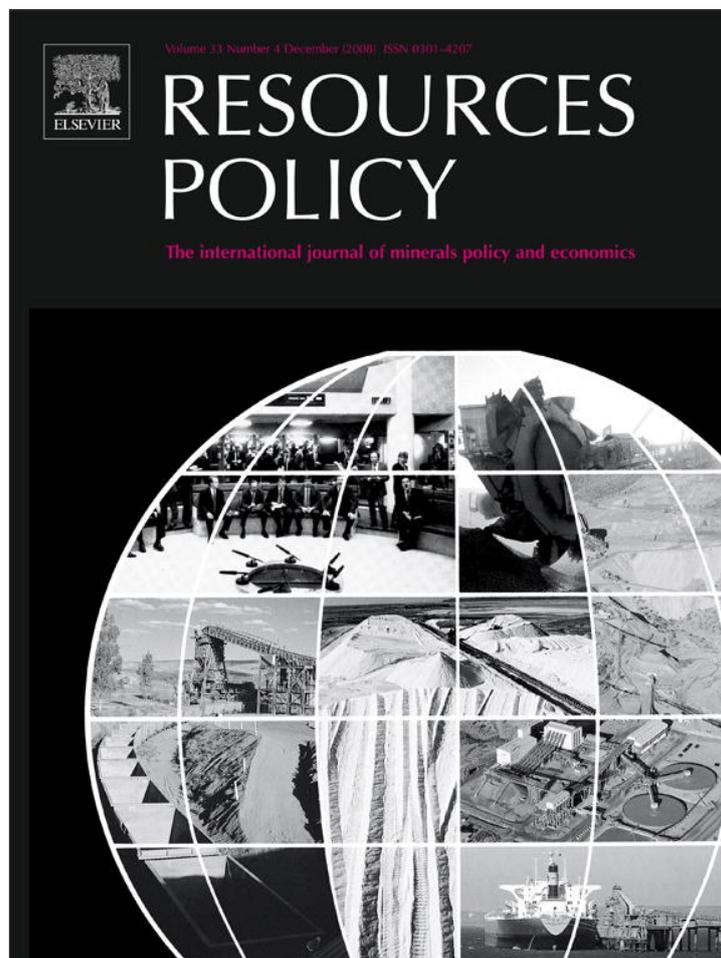


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## On technical change in the elasticities of resource inputs

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## ABSTRACT

This article analyses growth of an economy where the substitutability between non-renewable and renewable resource inputs changes over time. We allow for exogenous technical change in the elasticity of substitution (EoS) between these two types of resources as well as for biased factor-augmenting technical change. Our main results are: (1) sustained technical change in the EoS is enough to overcome resource constraints; (2) productivity-enhancing technical change is most beneficial when directed to the resource which is currently most important for production; (3) the speed of productivity-enhancing technical change is crucial for its usefulness to overcome resource constraints; (4) sustainability depends critically on the type of technical change.

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## Introduction

The sheer size of today's population coupled with its endless desires imposes a never known demand on the production of goods and services. The world's GDP has never been as high as now, and with the advent of China's and India's economic expansions, it seems clear that world production is bound to increase substantially. A noticeable problem is, however, that production depends on various inputs, some of which are known to be limited as well as non-renewable. This has raised concerns (also by major institutions such as RFF, World Bank) for the possibility of continuing production at today's levels in the future. As pointed out in a special issue of the Review of Economic Studies already in 1974, the ability to extend our current opportunities to future generations when production utilises limited non-renewable resources depends on a variety of factors, of which two have been singled out as the most crucial ones: substitutability and technical change.

Substitutability is vital because, as Dasgupta and Heal (1974) show, non-renewable resources are essential for production if

other inputs are poor substitutes for them. For example, insulation can reduce the amount of oil necessary for heating, but cannot be a perfect substitute for oil. Empirical evidence for low substitution possibilities is provided by Cleveland and Ruth (1997). If this is actually the case, this strand of research predicts a bleak future for generations to come.

Technical change, either exogenous as in Solow (1956) or endogenous as in Romer (1990), is another factor vital for further understanding of production processes. As an example, one could think of cars which are developed such that they can do the same mileage with less and less petrol. Equipped with these new growth-theoretic tools, resource economists (e.g. Scholz and Ziemes, 1999; Schou, 2000; Bretschger, 2005; Grimaud and Rouge, 2005) return with the hope that, even under initially unfavourable circumstances, production possibilities would not decline due to human ingenuity. These theories teach us that the general requirement for non-declining production is a fast enough technical change. However, it is again Cleveland and Ruth (1997, p. 217) who forcefully argue that "...technology and substitution have not been sufficiently strong to offset the effects of depletion at the macroeconomic scale in some nations".

Given the extensive theoretical research on a time-invariant elasticity of substitution (EoS) as well as on factor-neutral technical change on the one side, but the rather bleak outlook from the empirical literature on the other side, we shall consider a

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different way of investigating the potential for sustainability.<sup>1</sup> This we will do in the following way.

Unlike the classic works (Solow, 1974; Stiglitz, 1974; Dasgupta and Heal, 1974) which mainly analyse the relationship between capital and non-renewable resources, this article concentrates on the relationship between non-renewable and renewable resources in production. We use a production function of the constant elasticity of substitution (CES) type to allow for various degrees of substitutability between these two inputs. Thus, our paper is closely related to recent articles by André and Cerdá (2005) as well as Grimaud and Rouge (2005). Our analysis, though, unlike these contributions, allows for technical change in the actual EoS. This permits us to gain new insights into short-run and long-run dynamics. To our knowledge, this approach is novel to the literature.<sup>2</sup> In addition, we will compare such a framework with a case of biased factor-augmenting technical change.

The idea of technical change in the EoS does not just come out of nowhere. Demands to analyse the effect of technical change on the EoS have been raised time and again during the past 70 years, with stronger and clearer requests. Already Hicks (1932, p. 120), who invented the concept of the EoS in the 1930s, points out that the EoS might change because "(...) methods of production already known, but which did not pay previously, may come into use". Even more precise, he suggests that the increase in substitutability "(...) partly takes place by affording a stimulus to the invention of new types". de La Grandville (1989, p. 479) then proposes to think about the EoS as "a measure of the efficiency of the productive system", which is something that is going to become clear during the following sections. Yuhn (1991, p. 344), who tests de La Grandville's proposition, suggests to view the EoS as "a 'menu of choice' available to entrepreneurs". All of these researchers believe the EoS is an important component, if not a determinant, of growth. They furthermore suggest that the elasticity is by no means invariable over time. That there might exist technical innovations driving changes in the EoS has recently been put forward by Klump and Preissler (2000, p. 52) who suggest that "[a]s far as invention of new methods of production is concerned, the EoS as a measure of economic progress can, of course, be related to a society's capability to create and maintain a high rate of innovative activities". Finally, and most precisely, Bretschger (2005, p. 150) suggests that "all possibilities of substitution and, specifically, the effects technology exerts on promoting substitution have to be studied". Among others, these points quoted here provide a foundation for introducing technical change into the EoS.

In another section of this paper, we introduce technical change in the distribution parameters (and thus, unit productivities) of the renewable and non-renewable resources. We show how this can be linked to the literature on biased technical change (e.g. Acemoglu, 2003). We assume that both resource inputs are subject to technical change and therefore increase their productivity, but one of the resources is subject to quicker technical change than the other resource.<sup>3</sup> This allows us to compare our

results to those present in the literature, notably in the papers which adapt Acemoglu (2003) framework to renewable and non-renewable resources (e.g. Grimaud and Rouge, 2005) and in other related contributions (e.g. Amigues et al., 2006). To provide some slightly more intuitive perspective: if we take the Cobb–Douglas function as an example, then our extension permits to analyse the effect of changing Cobb–Douglas shares.

The ultimate objective of this paper is thus to analyse the implications of technological change in the elasticities of resource inputs for economic growth and sustainability. We wish to understand the conditions under which these types of technical change may lead to sustainability or even a positive long-run growth rate. This objective we approach via a theoretical growth model which allows us to study the long-run implications analytically, and to capture the detailed short-run dynamics numerically. We are interested in this since a variety of researchers have recently pointed at (and questioned) the possible importance of these kinds of technical change for growth and sustainability.

A further important article setting the stage for the current study is Tahvonen and Salo (2001) who show under historically plausible assumptions that in the course of economic development, the use of (energy) resources could evolve endogenously from renewables to non-renewables and back to renewables, even without any environmental policy. Tahvonen and Salo (2001) assume perfect substitutability between the two types of resources, however, which is presumably an overly optimistic assumption. By considering technical change which may, but may not, eventually lead to such perfect substitutability, our study can thus be viewed as an important qualification of their results and a bridge between their work and frameworks where a low EoS is assumed instead, such as Grimaud and Rouge (2005).

There are several reasons for which we focus on exogenous technical change here. Firstly, before trying to investigate under which circumstances a policy maker should invest more in the one type of technical change or the other, one has first to understand the generic dynamic effects of these changes. As we shall show, in many cases these generic effects are far from obvious. Secondly, the exogenous technical change approach enables one to implicitly recover the value a policy maker would attach to the increases in the EoS and other technological parameters. This helps predict how much should be invested in e.g. R&D aimed at increasing the EoS in more sophisticated environments.

The article is structured as follows. The second section lays out the simple growth framework which we use for studying the two types of technical change. The third section presents the benchmark case of no technical change. In the fourth section we introduce an increasing EoS. In the fifth section we compare these results to the results obtained when biased factor-augmenting technological change is allowed for. Policy implications are summarised in the sixth section. The seventh section concludes.

## The model

In the current section, we will first discuss technical change in the EoS and then—in the partial elasticities of non-renewable and renewable resources in the production function. Finally, we will lay out the setup of our model.

<sup>1</sup> By sustainability, we mean a strictly positive minimum level of income which should be achieved for all subsequent generations, i.e. for all  $t \geq 0$ .

<sup>2</sup> There are papers which consider an EoS which changes over time, but they do not refer to non-renewable and renewable resources and also, the mechanisms analysed therein do not have the "technological progress" flavour: the changes in EoS are modelled as by-products of other economic processes. These contributions are Miyagiwa and Papageorgiou (2007)—with a variable EoS between capital and labour, and Petith (2001)—with (broadly defined) land and labour.

<sup>3</sup> Alternatively, if factor-neutral technical change is absent or if the bias in technical change is stronger than its factor-neutral component, our modelling approach may imply that one of the resources increases its productivity while the other decreases it. Our main focus will remain with the case indicated in the main

(footnote continued)

text, but it should be remembered that our theoretical framework is equally well suited for accommodating any of these two cases.

Technical change in the CES function

Throughout the analysis, we shall be using the standard constant-returns-to-scale CES production function, as derived in the seminal article by Arrow et al. (1961). We shall allow for technical change in its EoS, or alternatively, in the distribution parameter of the CES function. The intermediate resource input  $R$  is produced with flows of non-renewable resource  $R_N$  and renewable resource  $R_R$ . They are combined according to

$$R(t) = [\psi(t)R_N(t)^{\theta(t)} + (1 - \psi(t))R_R(t)^{\theta(t)}]^{1/\theta(t)}, \tag{1}$$

where the distribution parameter is given by  $\psi(t) \in [0, 1]$ , and the EoS,  $\sigma(t) \in (0, +\infty)$ , is related to the elasticity parameter  $\theta(t) \in (-\infty, 1)$  via

$$\theta(t) = \frac{\sigma(t) - 1}{\sigma(t)}. \tag{2}$$

The EoS  $\sigma(t)$  and the distribution parameter  $\psi(t) \in [0, 1]$  are allowed to change over time. The non-renewable resource stock  $0 < S_N(0) < \infty$  is extracted according to  $\dot{S}_N(t) = -R_N(t) \leq 0$ .<sup>4</sup>

For the renewable resource  $R_R$ , we assume that it arrives in constant flows over time,  $R_R \equiv \text{const}$ . This is a strong assumption which helps simplify the subsequent analysis. Yet, it is excusable: Dasgupta and Heal (1974, p. 19) consider the renewable resource to be “a perfectly durable commodity which provides a flow of services at [a] constant rate”. Moreover, the renewable resource is a potential backstop technology<sup>5</sup>: it becomes a backstop technology if the economy succeeds in shifting  $\sigma$  above unity.

Taking the growth rate of  $R$  (and dropping time subscripts for simplicity) yields

$$\hat{R} = \varepsilon_N \hat{R}_N + \varepsilon_R \hat{R}_R + \varepsilon_\sigma \hat{\sigma} + \varepsilon_\psi \hat{\psi}, \tag{3}$$

where the partial elasticities,  $\varepsilon_i = (\partial R / \partial i)(i/R)$ , for  $i = R_N, R_R, \sigma, \psi$ , are given by

$$\begin{aligned} \varepsilon_N &= \frac{\psi R_N^\theta}{R^\theta}, \\ \varepsilon_R &= \frac{(1 - \psi) R_R^\theta}{R^\theta}, \\ \varepsilon_\sigma &= \frac{\psi R_N^\theta \ln R_N + (1 - \psi) R_R^\theta \ln R_R - R^\theta \ln R}{(\sigma - 1) R^\theta}, \\ \varepsilon_\psi &= \frac{\psi (R_N^\theta - R_R^\theta)}{\theta R^\theta}. \end{aligned}$$

Both  $\varepsilon_N \in [0, 1]$  and  $\varepsilon_R \in [0, 1]$  function as shares in the traditional sense. This is due to the assumption of constant returns to scale in the CES function. It can be shown that  $\varepsilon_\sigma > 0$  which implies that improvements in the EoS always increase output. We also have that  $\varepsilon_\psi > 0$  if  $R_N > R_R$  ( $\varepsilon_\psi < 0$  if  $R_N < R_R$ ) suggesting that technical change in the distribution parameter is most useful when it goes to the resource that is currently more important for production.

A definitional note must be made here: throughout the paper, *more important for production* means *used in larger amounts* if the resources are gross substitutes ( $\sigma \geq 1$ ), and *used in smaller amounts* if the resources are gross complements ( $\sigma < 1$ ). This complication stems from the fact that the distribution parameter  $\psi$  is related to the unit productivity of non-renewable resource  $R_N$  (denoted as  $a$ ) via  $\psi = a^\theta$ . Their mutual relationship is thus positive if and only if

$\theta > 0$  (that is,  $\sigma > 1$ ). Please refer to this definition wherever we use the phrase “more important for production”.

This result is complementary to the one obtained by André and Cerdá (2005): optimising subject to both resource inputs allows them to find a corner solution where it might be useful to delay the exploitation of the renewable resource in order to allow for its stock to grow towards the maximum sustainable yield. Such result is not possible here, as we concentrate on the case with constant renewable resource flows. However, we are more precise in defining under which circumstances it is more profitable (in terms of total output) to increase the relative productivity of a given type of resource: if the two types of resources are gross substitutes, it should be used in relatively larger amounts; if they are gross complements, it should be used in relatively smaller amounts. This result is new to the literature and somewhat surprising. Usually, the focus is on technical change directed towards the resource whose input in production is currently smallest, which is supposed to compensate the reductions in resource usage with higher unit productivities. However, our simple growth accounting exercise suggests that this is the case only if  $\sigma < 1$ . We shall confirm in the subsequent sections that our result holds both over the long run and within the transitory dynamics.

CES and changing EoS

The EoS  $\sigma$  represents the percentage change in relative quantities of used resources following a one percent change in their relative prices. As  $\sigma \rightarrow 0_+$  (i.e.  $\theta \rightarrow -\infty$ ) then the function converges to the Leontief function where the inputs are perfect complements (thus, for  $\sigma = 0$ , the formula (1) should be replaced with an appropriate Leontief function); for  $\sigma \rightarrow 1$ , the standard Cobb–Douglas form is converged to, and for  $\sigma \rightarrow +\infty$ , the function becomes linear in the limit, implying that the inputs are perfect substitutes. The EoS therefore gives information on the ease with which one can move along a given isoquant, and in that way it can be understood either as a measure of flexibility, efficiency (de La Grandville, 1989), or “menu of choice” (Yuhn, 1991). We shall view the EoS as a measure of technical efficiency.<sup>6</sup>

As can be seen in Fig. 1, the higher the EoS, the larger the amount of output  $R$  for any given combination of inputs  $R_N$  and  $R_R$ . Even small improvements in the EoS have a positive effect on total output production. Furthermore,  $R$  as a function of  $\sigma$  follows a convex–concave shape.

The EoS is not only a measure of efficiency, though. It pulls double duty, since it also determines the essentiality of particular inputs. If  $\sigma \leq 1$ , then  $R \rightarrow 0$  if any of the two inputs goes to zero. On the contrary, if  $\sigma > 1$ , neither of the inputs is essential any longer.

We shall assume exogenous technical change in the EoS, as summarised by the following reference formula:

$$\sigma(t) = \sigma(0) e^{\sigma t}, \quad \sigma(0) > 0, \tag{4}$$

which implies that the growth rate of  $\sigma$  is constant. Moreover, whatever the initial relationship between our inputs, they will become perfect substitutes in the limit. This assumption simplifies the subsequent analysis a lot but is admittedly strong, and in fact not necessary: if we take the EoS as a “dummy” for the essentiality of inputs then we only require  $\sigma$  to exceed unity from some point in time on, and our results will be preserved. But, as is

<sup>4</sup> Throughout the article we shall use  $\dot{B}(t)$  to denote the time derivative of  $B(t)$  and  $\hat{B}(t)$  to denote its growth rate. For any function  $G(B)$ ,  $G'(B)$  denotes its first derivative, and  $G''(B)$  its second derivative with respect to  $B$ .

<sup>5</sup> Nordhaus (1973, pp. 547–548) writes: “The concept (...) is the backstop technology, a set of processes that (1) is capable of meeting the demand requirements and (2) has a virtually infinite resource base”.

<sup>6</sup> For example, some time ago, if one wanted to produce shoes, one needed both leather and rubber. Imagine that one had an abundance of rubber, and leather enough for just one shoe. Then, one could only use up all leather, produce a single shoe, and waste all remaining rubber. Clearly, this was not very efficient. Now as we can produce shoes with more flexibility, one may substitute rubber for scarce leather (or vice versa) and thus make many more shoes and not waste the rest of the rubber.

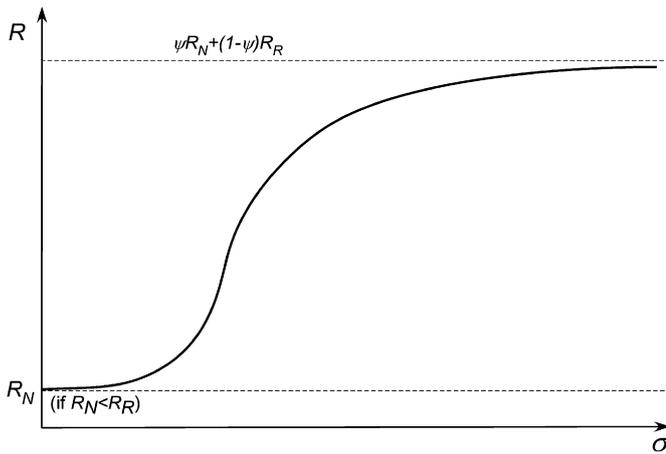


Fig. 1.  $R$  as a function of  $\sigma$ .

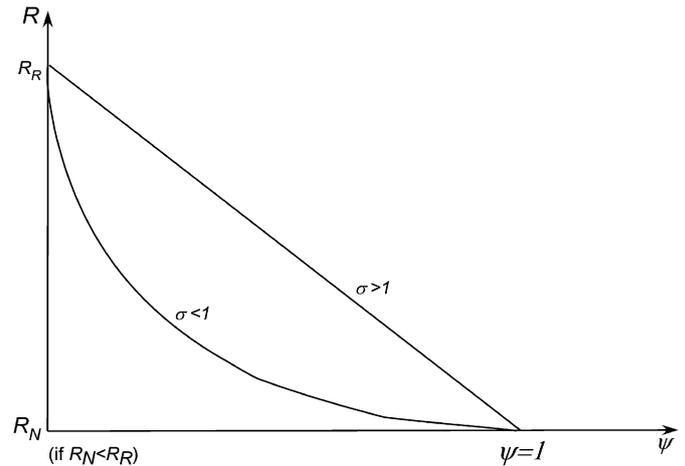


Fig. 2.  $R$  as a function of  $\psi$ .

obvious from Fig. 1, the consecutive efficiency gains from improvements in the EoS should not be neglected. We shall see shortly that these gains provide new qualitative insights for the short-run dynamics. Finally, allowing the EoS to approach infinity in the limit allows our article to serve as a bridge to Tahvonen and Salo (2001) who assume perfect substitutability between non-renewable and renewable resource inputs.<sup>7</sup>

*CES and biased technical change*

As an alternative to a changing EoS, we shall introduce technical change in the distribution parameter  $\psi$  which proxies the bias in factor-augmenting technical change. We shall assume that technical change affects the relative share of the two inputs by changing the distribution factor as follows:

$$\psi(t) = \psi(0)e^{zt}, \quad \psi(t) \in [0, 1], \quad \psi(0) \in (0, 1), \quad (5)$$

where the growth rate of  $\psi$  is a constant  $z \leq 0$ . Again, the presumed exponentiality of decay in  $\psi$  is an assumption of convenience rather than a result which could be tested empirically. It is however frequently met in the theoretical literature (see e.g. Grimaud and Rouge, 2005).

The effect of changing  $\psi$  on  $R$  is

$$\frac{\partial R}{\partial \psi} = \frac{1}{\theta} (R_N^\theta - R_R^\theta) [\psi R_N^\theta + (1 - \psi) R_R^\theta]^{(1-\theta)/\theta}, \quad (6)$$

implying that  $\partial R / \partial \psi > 0$  if  $R_N > R_R$ , and  $\partial R / \partial \psi < 0$  if  $R_N < R_R$ . Fig. 2 illustrates the effect of changing  $\psi$  for the case of  $R_N < R_R$  and two different values of  $\sigma$ . The lower the possibility to substitute the stronger the initial effect of changes in  $\psi$ . In the extreme, for Leontief inputs, a marginal change from  $\psi = 0$  to  $\psi > 0$  will result in a reduction in the intermediate output from  $R_R$  to  $R_N$  (given  $R_N < R_R$ ). Biased technical change can thus be viewed as varying the relative productivity of  $R_R$  and  $R_N$ .

The marginal rate of technical substitution is given by  $MRTS = (\psi / (1 - \psi)) (R_N / R_R)^{-1/\sigma}$ : changes in  $\psi$  affect the slope of the isoquants of  $R$  leaving their curvature intact.

*The growth model*

We shall now embed our intermediate resource input  $R$  with technical change in  $\sigma$  or  $\psi$  in a simple Ramsey-type model with an

infinite planning horizon, where a representative agent maximises discounted utility subject to an equation of motion of the non-renewable resource stock.

Formally, this means that our infinitely lived representative agent maximises

$$\begin{aligned} \max_{\{R_N(t)\}_{t=0}^{\infty}} \int_0^{\infty} U(Y(t)) e^{-\rho t} dt, \\ \text{subject to } \dot{S}_N(t) = -R_N(t), \\ R_R \equiv \text{const}, \\ Y(t) = A(t)R(t)^\beta, \\ R(t) = [\psi(t)R_N(t)^{\theta(t)} + (1 - \psi(t))R_R^{\theta(t)}]^{1/\theta(t)}. \end{aligned}$$

At this point we assume that  $U(Y) = Y^{1-\gamma} / (1 - \gamma)$ , where  $\gamma = -YU''(Y) / U'(Y) \in (0, \infty)$ <sup>8</sup> is the inverse of the (constant) intertemporal EoS.

Technical change is decomposed into three components: factor-neutral technical change  $A(t) = A(0)e^{gt}$  with  $g \geq 0$ , technical change in the EoS,  $\sigma(t) = \sigma(0)e^{st}$  with  $s \geq 0$ , and biased technical change due to  $\psi(t) = \psi(0)e^{zt}$  with  $z \leq 0$ . On top of this, we assume that either  $s = 0$  with  $z < 0$  or  $z = 0$  with  $s > 0$  for the sakes of normalisation<sup>9</sup> and transparency of results.

Since no savings decision is allowed for, all output is immediately consumed and thus  $C(t) = Y(t)$  for all  $t$ .

The simple, “bare-bones” model which we use features a number of strong simplifying assumptions, such as constancy of the renewable resource flow over time, exogenous technical change, and the neglect of capital accumulation. We agree to pay such a price because in return we get a clear understanding of the dynamics in all, even the most complex cases. This makes our article a good starting point for further analyses: relaxing several of the assumptions would provide interesting questions for future research. In particular, we point out that our results should be verified in an optimal growth framework which fully endogenises research in both resource inputs. Also the mechanisms behind technical change in our basic model should be provided with a formal treatment.

<sup>8</sup>  $\gamma > 1$  implies that utility is bounded from above.  $\gamma \geq 1$  implies that the utility of zero consumption is minus infinity, enforcing positive consumption at all times. If  $\gamma < 1$  then zero consumption at some (later)  $t$  is not automatically ruled out.

<sup>9</sup> A proof that our approach is consistent with normalisation (cf. de La Grandville, 1989; Klump and Preissler, 2000) is available from the authors. These issues have been neglected in most prior literature but are clearly important if one does not want to do inter-family comparisons of CES functions.

<sup>7</sup> For a more detailed discussion of the possible interpretations as well as of the micro-economic background of technical change in the EoS, please consult the working paper version of this article, Growiec and Schumacher (2006).

The first-order condition gives us the Ramsey–Hotelling condition which characterises the interior solution of the optimisation problem. After rearranging, we obtain that

$$\hat{Y} = \frac{\hat{F}_N - \rho}{\gamma}, \tag{7}$$

with  $F_N = \partial Y / \partial R_N$ , which states that the growth rate of income,  $\hat{Y}$ , is positive if and only if the growth rate of the marginal product of the non-renewable resource,  $\hat{F}_N$ , exceeds the discount rate.

In its core, this result is similar to the outcome of the “cake-eating” model (see e.g. Dasgupta and Heal, 1974). In the cake-eating model, the marginal product of the non-renewable resource is a function of two variables only: the rate of depletion and the rate of factor-neutral technical change. Hence, without large enough factor-neutral technical change, the growth rate of the marginal product of the non-renewable resource will be negative for all times and income will decline continuously. Analogously, income growth can be positive in our “Ramsey–Hotelling” model only if technical change is fast enough to keep the growth rate of the marginal product of the non-renewable resource above the discount rate.

As we decompose technical change into three different kinds, we obtain more possibilities to achieve sustainable production. The way in which Dasgupta and Heal (1974) results are changed due to the introduction of technical change in the EoS and biased technical change through the distribution parameter will become clear in the subsequent sections.

Solving Eq. (7) for  $\hat{R}_N$ , we obtain the optimal growth rate of the non-renewable resource extraction in terms of other variables:

$$\hat{R}_N = \frac{(1-\gamma)g - \rho + [(1-\gamma)\beta - \theta]\varepsilon_\sigma - (1/\sigma)(\ln R - \ln R_N)s + [(1-\gamma)\beta - \theta]\varepsilon_\psi + 1z}{(1-\theta) - ((1-\gamma)\beta - \theta)\varepsilon_N}. \tag{8}$$

As proved in the second part of the Appendix, the denominator of the above expression is always positive. We also notice that all three kinds of technical change affect the dynamic path of the optimal resource extraction rate  $R_N$ .

### Comparative statics

Comparative statics of our model provide further insights into the impact of its certain parameters on the growth rate of income.<sup>10</sup>

Along the optimal path, the discount rate affects the growth rate of income always negatively. Clearly, the less we care about the future the more resources we use up now. This brings about a larger current level of income but less growth potential since less resources are left for use later on.

The growth rate of income is unambiguously positively related to the rate of factor-neutral technical change  $g$ . This result also carries forward from established literature.

We have already shown that changes in the distribution factor should always go towards the kind of resource which is more important for production. Increases in  $z$  have a positive effect of income growth if  $\varepsilon_{\psi} > -\varepsilon_N / (1 - \theta)$ . Hence, a sufficient condition for a positive effect of  $z$  on income growth is  $\varepsilon_{\psi} > 0$ , i.e. if the resource which is more important for production is also subject to relatively faster technical change.

The effect of  $s$  on income growth depends on whether more renewable or more non-renewable inputs are used in production. If more non-renewable inputs are used then the effect of  $s$  on income growth is unambiguously positive. If more renewable resources are used, then the effect of  $s$  on income growth depends

on the importance of the non-renewable resource for production relative to the partial elasticity of  $\sigma$  and cannot be unambiguously signed. This observation is then crucial for the transition period, pointing at the significance of looking at short-run consequences of EoS growth.

### The benchmark case

#### The long run

The properties of the benchmark model with no technical change in  $\sigma$  ( $s = 0$ ) and  $\psi$  ( $z = 0$ ) are similar to the ones established in previous literature. Two important sub-cases should be distinguished here:  $\sigma > 1$  and  $\sigma \leq 1$ . These cases have already been presented in the literature (see e.g. Dasgupta and Heal, 1974). We shall quickly go through them here since they provide the benchmark to which our subsequent extensions will be compared.<sup>11</sup>

If non-renewable and renewable resources are gross substitutes,  $\sigma > 1$ , then the non-renewable resource is not essential for production. It becomes a virtually negligible input when it approaches zero. Hence, income growth can stay positive forever if only there is factor-neutral technical change,  $g > 0$ .

If the two types of resources are gross complements ( $\sigma \leq 1$ ), we have that the partial elasticity of the non-renewable resource tends to one because  $R_N$ , as it approaches zero, determines output. In the case  $\sigma < 1$ , the rate of resource depletion provides a stronger drag on per capita income than in the Cobb–Douglas case ( $\sigma = 1$ ).

As far as long-run results are concerned, the benchmark case confirms the decisive importance of the EoS in the production process, a result that carries forward from the classical literature.<sup>12</sup>

#### The short run

We study the optimal path of income in the benchmark case using numerical simulations. The results (with  $g = 0$ ) are presented in Fig. 3.<sup>13</sup>

In the case  $\sigma > 1$ , there is finite-time depletion of  $R_N$  but it happens late enough not to affect the short-run results substantially.<sup>14</sup>

The interpretation of these results is as follows. The discounted utilitarian criterion regards future consumption as less important than current consumption, wherefore most non-renewable resources will be extracted when utility seems most valuable. The simulations show that in case the non-renewable resource is not essential for production ( $\sigma = 2$ ), the share of  $R_N$  in production is initially very large, suggesting that most non-renewable resources are used initially, and then declines to zero over time. Thus the level of GDP is initially larger for the case of high EoS in comparison to the case of  $\sigma \leq 1$ , as technical efficiency does not

<sup>11</sup> Readers interested in the mathematical derivation of these results may consult the working paper version of this article, Growiec and Schumacher (2006).

<sup>12</sup> The important transversality condition, guaranteeing finiteness of the objective integral, is for all sub-cases  $(1 - \gamma)g < \rho$ . It is automatically satisfied if  $\gamma > 1$  and  $g \geq 0$ .

<sup>13</sup> The simulations were run in Matlab. Unless stated otherwise, we used the following parameter choices in all simulations:  $\beta = \frac{1}{3}$ ,  $\gamma = 2$ ,  $R_R = 2$ ,  $\rho = 0.05$ ,  $\sigma(0) = 0.1$ ,  $\psi(0) = 0.9$ ,  $S(0) = 300$ .

<sup>14</sup> Finite-time depletion obtains also for the case where  $\gamma < 1$  (so that zero consumption is not penalised by infinite disutility), regardless of the magnitude of EoS. Throughout our numerical exercises, we assumed  $\gamma > 1$ , however, thereby ruling out paths with zero consumption at some time  $t$ . For a more thorough discussion of the possibility of finite-time depletion, please refer to the working paper version of this article, Growiec and Schumacher (2006).

<sup>10</sup> Derivations are available in the working paper version of this article, Growiec and Schumacher (2006).

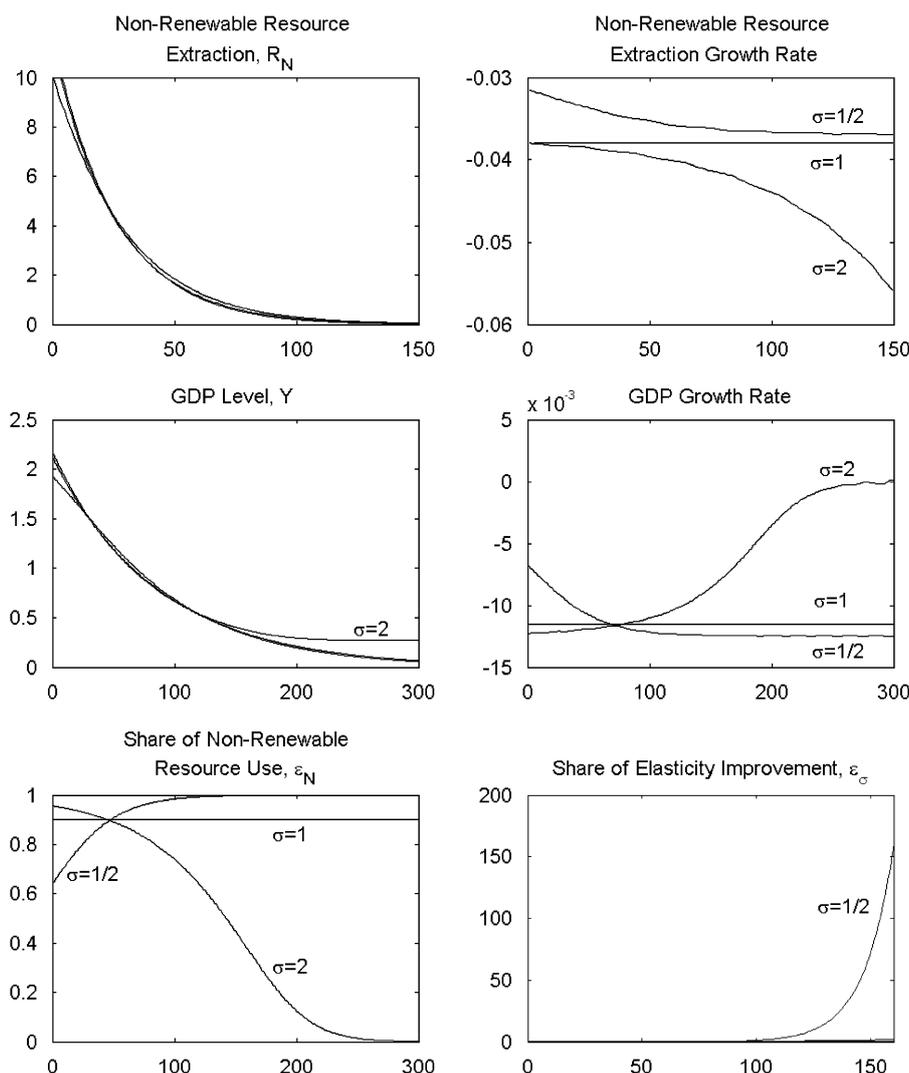


Fig. 3. Benchmark case,  $s = 0, z = 0, g = 0$ .

constrain the intermediate input  $R$  by complementarity problems. The (uniformly negative since  $g = 0$ ) GDP growth rate is initially higher in the case of complementarity as at first more non-renewable resources are conserved because production will be depending on those resources later. Moreover, the amount of the non-renewable resource used in production is divided more equally over time the lower is  $\sigma$ , due to its essentiality for production.<sup>15</sup> Nevertheless though, if the non-renewable resource and the renewable one are gross complements in production, a decrease in the amount of non-renewable resources available decreases the overall resource bundle that can finally be used in production. In such case, the non-renewable resource is driving the size of the resource bundle  $R$ , implying that the share of the non-renewable resource in the resource bundle will approach one. Hence, the (negative) GDP growth rate will become smaller and smaller. In contrast to this, if the non-renewable resource is inessential ( $\sigma > 1$ ) then, as less and less of it remains available, its share in the resource bundle will drop to zero. In this sense, the non-renewable resource allows greater GDP levels as long as it is available, but as soon as it gets depleted, GDP goes to the level that

it would have had, had it been produced without the non-renewable resource from the outset.

### Increasing flexibility

#### The long run

Let us now analyse the effects of allowing for technical change in the EoS  $\sigma$ , but not in the distribution parameter  $\psi$ , such that  $s > 0, z = 0$ . The long-run asymptotics will be exactly the same as in the benchmark case, sub-case  $\sigma > 1$ . Hence, for the qualitative features of the long run, there is no need to assume unbounded growth in the EoS. We would have obtained the same asymptotic GDP growth rate if we had assumed that  $\sigma$  grew only until some time  $t_0$ , at which  $\sigma(t) > 1$ , i.e. non-renewable resources were not essential for production any more. Hence, these asymptotic properties hold as long as the EoS manages to cross the “magical” barrier of one.

#### The short run

In our “increasing flexibility” case we know that from a certain time  $t_0 \geq 0$  onwards, the EoS will exceed one, and thus both kinds

<sup>15</sup> For the case  $\sigma = 2$ ,  $R_N$  gets depleted around  $T^* = 160$  which is not visible to the naked eye.

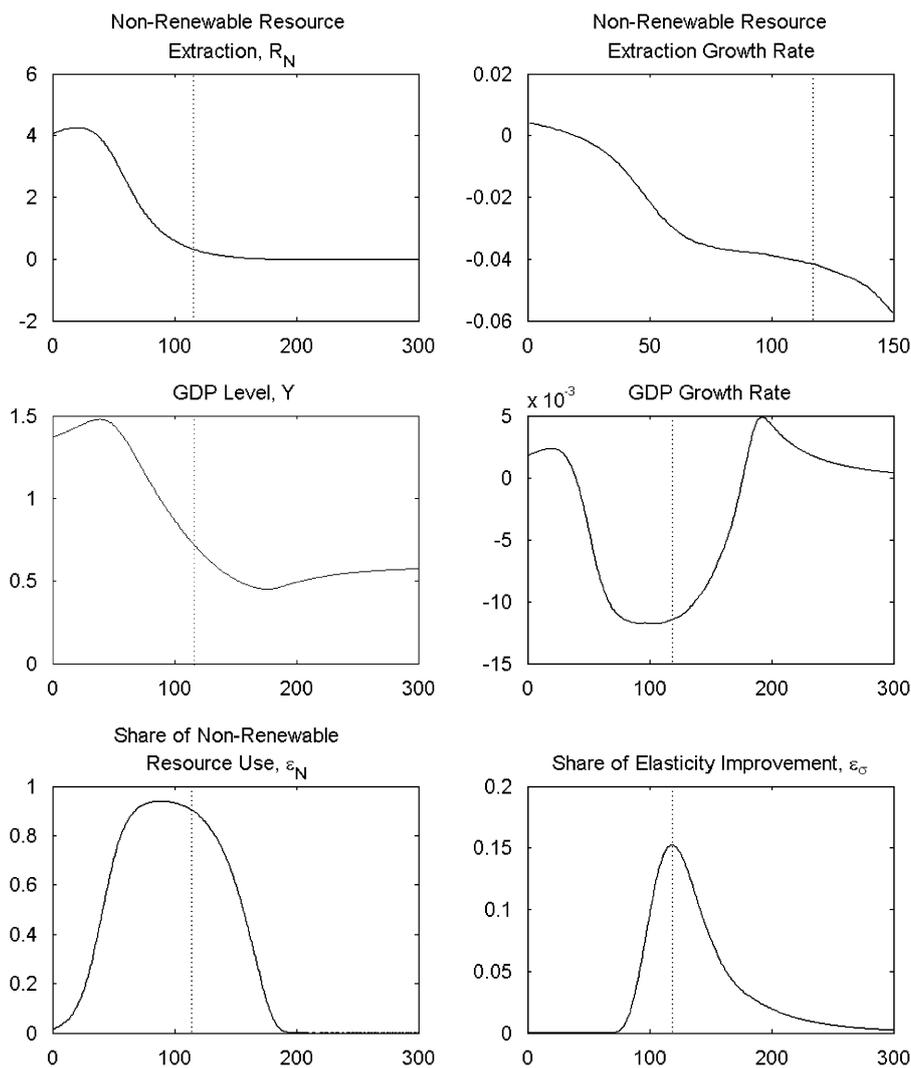


Fig. 4. Increasing flexibility case,  $s = 0.02$ ,  $z = 0$ ,  $g = 0$ .

of resource inputs will be gross substitutes. The non-renewable resource will then necessarily be depleted in finite time. This result is intuitive because perfect substitutability between non-renewable and renewable resources allows us to utilise the non-renewable resource without compromising the productivity of the renewable one in any way.

The results of the simulative exercise (with  $g = 0$ ) are presented in Fig. 4. Vertical dashed lines indicate the moment in which the EoS crosses one.

As the EoS increases it is at first optimal to utilise non-renewable resources at an increasing rate. This is because initially, the improvements in technical efficiency allow for large increases in the intermediate input  $R$ .<sup>16</sup> Hence, this allows for an increasing growth rate of income. However, these initial exponential-like increases in  $R$ , due to the increasing EoS, level off quickly, such that consecutive improvements in  $\sigma$  increase the overall resource bundle only slightly. It is now optimal to slow down the extraction of the non-renewable resource, as it is still essential for production. At this time, the model is rather similar to the benchmark case with an essential resource and produces a

negative growth rate of income. At one point in time, though, the technical efficiency will have improved substantially enough, so that the non-renewable resource will not be essential for production any more. The model will then trace the benchmark case again, but with a non-essential non-renewable resource. However, as continued improvements in technical efficiency feed into the production process, the economic growth rate can be positive. As the consecutive improvements in efficiency increase production by less and less (the effect of  $\sigma$  is bounded), and as the non-renewable resource stock is depleted in some finite time after  $\sigma$  has crossed unity ( $T^* > t_0$ ; in the simulated case, we obtained  $T^* = 170$ ),<sup>17</sup> the growth rate of income tends to zero from above.

Improvements in the EoS pull double duty: on the one hand, the EoS reflects technical efficiency, and on the other hand, it reflects the essentiality of exhaustible resources in production. We find that improvements in technical efficiency are more relevant for the short run, whereas all that matters in the long run is essentiality.

<sup>16</sup> The initial level of  $\sigma(t)$  is set at  $\sigma(0) = 0.1$ , which corresponds to the strongly increasing part of Fig. 1.

<sup>17</sup> Again, this fact is not visible to the naked eye in the figure.  $R_N$  approaches zero smoothly and approaches the vicinity of zero much earlier than it actually takes the zero value.

## Biased technical change

### The long run

We shall now compare the case of technical change in the EoS with the case of biased technical change. We assume  $s = 0$ ,  $z < 0$ , and  $g \geq 0$  which implies that: (i) if  $\sigma > 1$ , technical change improves the efficiency of the renewable resource more quickly than it improves the efficiency of the non-renewable resource, (ii) if  $\sigma < 1$ , the direction is reversed, (iii) if  $\sigma = 1$ , technical change gradually reduces the Cobb–Douglas share of non-renewable resources. Having Cleveland and Ruth (1997) observation in mind, namely that technical change has not been sufficiently strong to offset the effects of depletion, we believe that the possibility of technical change in the distribution parameter deserves careful scrutiny. In this section, we shall explain why and when this kind of biased technical change might be useful.

We shall be dealing with three sub-cases:  $\sigma > 1$ ,  $\sigma = 1$ , and  $\sigma < 1$ , where the last case is further divided into three sub-sub-cases of fast, medium, and slow biased technical change, delineated by the expression  $z - \theta(\rho - (1 - \gamma)g)$  being negative, zero, or positive, respectively.

*Case  $\sigma > 1$ .* If the resource inputs are gross substitutes, then as the renewable resource can be easily substituted for the non-renewable resource, its flow into the intermediate resource input alone can guarantee positive long-run output. In this case, biased technical change has no qualitative effect on the asymptotic results and the non-renewable resource will be depleted in finite time just as in the benchmark case.

*Case  $\sigma = 1$ .* For the Cobb–Douglas case, it turns out that biased technical change where the share of the non-renewable resource  $\varepsilon_N = \psi \rightarrow 0$  is enough to guarantee positive output forever even in the absence of factor-neutral technical change ( $g = 0$ ). In contrast to the  $\sigma > 1$  case,  $R_N$  will be depleted only in infinite time.

*Case  $\sigma < 1$ .* If the resource inputs are gross complements, the speed of technical change is crucial for the long-run results. We find that three distinct regimes may emerge. In all of them, though, only infinite-time depletion is possible.

*Fast technical change:*  $z < \theta(\rho - (1 - \gamma)g)$ . This assumption implies that non-renewable resource-augmenting technical change is quick enough to fully compensate for the declining flows of these resources. This condition is more likely to be satisfied the weaker the complementarity of resource inputs (the greater the negative  $\theta$ ). Analogously, the greater the degree of complementarity between the resource inputs, the faster must be the increase in the relative productivity of the renewable resource.

*Medium technical change:*  $z = \theta(\rho - (1 - \gamma)g)$ . In this knife-edge case, the speed of technical change is just enough to guarantee that the share of the non-renewable resource,  $\varepsilon_N \rightarrow \bar{c}$  where  $\bar{c} \in (0, 1)$ . The depletion of  $R_N$  provides a drag on the long-run growth rate of the economy. These results are similar to the less parsimonious case of slow technical change, described below. The details are available from the authors upon request.

*Slow technical change:*  $z > \theta(\rho - (1 - \gamma)g)$ . If technical change is too slow to fully compensate for the shrinking flow of  $R_N$ , then its depletion exerts an unambiguously harmful effect on the long-run growth rate of the economy.

These results allow us to draw conclusions which are a little more precise than those of recent research. For example, André and Cerdá (2005) derive equations describing the optimal dynamic evolution of the resource input ratio and the optimal output path, and then conclude from these that the “equations (...) express, in a mathematical way, the interest (and, in the long run, the need) to promote the research and use of renewable energy sources (...) to substitute nonrenewable energies (...) from a sustainability perspective”. Our analysis allows to find that biased

technical change has its merits when it is directed to the resource input which is *most important for production*. In fact, in the short run it might turn out optimal to invest more in research promoting the other type of resource than in the long run.

One more finding should be emphasised here. In the case  $\sigma < 1$ , there exists a “bifurcation” set of parameter values, which bounds away from the other cases of qualitatively different dynamic behaviour of the model. Indeed, authors working within the (Acemoglu, 2003) framework deal with (endogenously determined) biased technical change of the type which we call “slow”; their models typically do not allow for a jump to the regime where biased technical change is “fast”.<sup>18</sup>

Summarising, the long-run results for the biased technical change case stand in stark contrast to the results for the increasing flexibility case where technical change was unambiguously good and its desirable direction could be only towards higher substitutability. Moreover, we find here that the actual *value* of substitutability  $\sigma$  plays a more important role for the long-run dynamics even than the *growth rate* of the distribution parameter  $\psi$ , denoted as  $z$ . A one-shot improvement in the EoS could be more beneficial for the economy than perpetual growth in the unit productivities of resource inputs.

### The short run

The results of the simulative exercise for the case of biased technical change (with  $g = 0$ ) are presented in Figs. 5 and 6. In the first of these figures, we distinguish between the three major cases of  $\sigma > 1$ ,  $\sigma = 1$  and  $\sigma < 1$ . In the second one, we analyse the difference in dynamics stemming from the fact that the speed of biased technical change  $z$  may be either slow or fast (in Fig. 5, it is slow).

We can interpret these simulation results as follows. The discounted utilitarian criterion leads the infinitely lived agent to perceive her current income as more important for the creation of utility, so she initially utilises most non-renewable resources. As in all previous cases, we observe the crucial role played by the EoS. If  $\sigma < 1$  then the production is driven by the resource which is used in relatively smaller amounts. Conversely, if  $\sigma > 1$ , then the relatively more heavily used resource is decisive. Initially, more non-renewable resources are utilised in production. For the case of  $\sigma < 1$ , this implies that the renewable resource constrains output, and therefore relatively quicker improvements in the marginal product of the non-renewable resource (as captured by  $z < 0$ ) are not effective and cannot reduce this drag. When  $\sigma > 1$ , then  $z < 0$  captures technical change biased towards the renewable resource which is again ineffective because now it is the relatively more heavily used non-renewable resources which drive output. In both cases the effect on GDP growth is negative.

The situation is reversed once the non-renewable resources start being used for production in relatively smaller amounts. In such case, prevalent over the long run, technical change captured by  $z < 0$  is uniformly beneficial and helps reduce the drag on GDP growth accruing from resource depletion. There is one additional effect here which can be observed in the plot of  $\varepsilon_N$ . If  $\sigma < 1$  then the share of non-renewable resources in production rises towards unity, indicating that these resources are significant constraint to production. If  $\sigma \geq 1$ , this share falls to zero so that this constraint is alleviated. A further qualification to this result can be found in Fig. 6: this constraint can be alleviated even if  $\sigma < 1$ , provided that technological change is fast enough.

<sup>18</sup> To keep our results close to this strand of literature, one should redefine  $\psi \equiv \hat{a}^\theta$ . Then,  $z = \theta \hat{a}$  with  $\hat{a} > 0$ , and the condition for “fast” biased technical change becomes  $\hat{a} > \rho - (1 - \gamma)g$ .

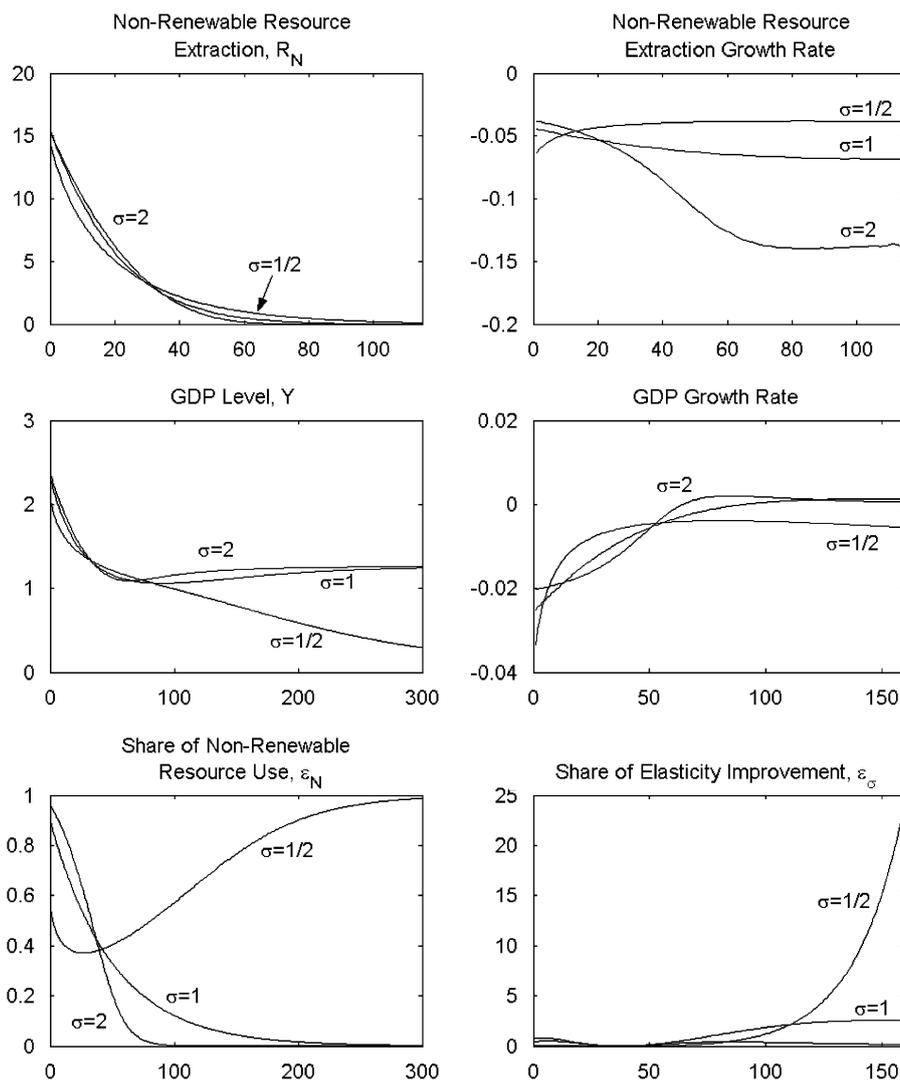


Fig. 5. Biased technical change case,  $s = 0, z = -0.02, g = 0$ .

We also see that  $\varepsilon_\sigma$  explodes to infinity in the  $\sigma < 1$  case, indicating that improvements in the EoS would be increasingly valued as  $R_N$  gets depleted.

Further simulations show that in case the resources are gross complements ( $\sigma < 1$ ), the speed of  $z$  is vital in order to allow for a non-decreasing long-run income. For the case of  $\sigma = \frac{1}{2}$ , our simulations demonstrate that the assumed speed of biased technical change,  $z = -0.02$ , is not sufficient to outweigh the decreases in non-renewable resource flows. Hence, the GDP level will fall to zero, a result in line with the benchmark case, sub-case  $\sigma \leq 1$ . However, if the speed of  $z$  outweighs the reduction in the flow of non-renewable resources (in our case,  $z = -0.06$ ), then a positive constant GDP level is attainable. The speed of  $z$  must be such that the renewable resource becomes more and more important in production quickly enough to compensate for the decreasing quantities of the non-renewable resource available,  $z < \theta(\rho - (1 - \gamma)g)$ . Only then is it guaranteed that technical change wins the race against the gradual disappearance of the essential production input.

### Policy implications

The results outlined above have a number of interesting implications which could be useful for policy purposes. Four

principal policy-oriented corollaries from our analysis are discussed below.

**Corollary 1.** *It is reasonable to assume that, at least as for today and for the macroeconomic scale, non-renewable resources are essential for production. It is hard to imagine production without non-renewable resources, and even though sometimes we can already substitute renewable resources for non-renewable ones, like biodiesel for standard diesel, currently available substitution possibilities are visibly limited. Furthermore, biodiesel might be less efficient than standard diesel and not all diesel engines might drive on biodiesel without sufficient, costly adjustments. Hence, the question would be: should a policy maker invest to improve the substitutability and, if so, for what reasons?*

Our analysis sheds light on three issues relevant to this question. Firstly, we have reproduced the standard result that if renewable resources were a “true” (gross) substitute for non-renewable resources on the macroeconomic scale, then production possibilities would not be critically dependent on the use of non-renewable inputs. Secondly, we have shown that higher substitutability increases productive efficiency. Thus, improvements in the substitutability are useful even if the inputs are not essential any longer. In such case, even small increases in substitutability may allow for positive output growth. Thirdly

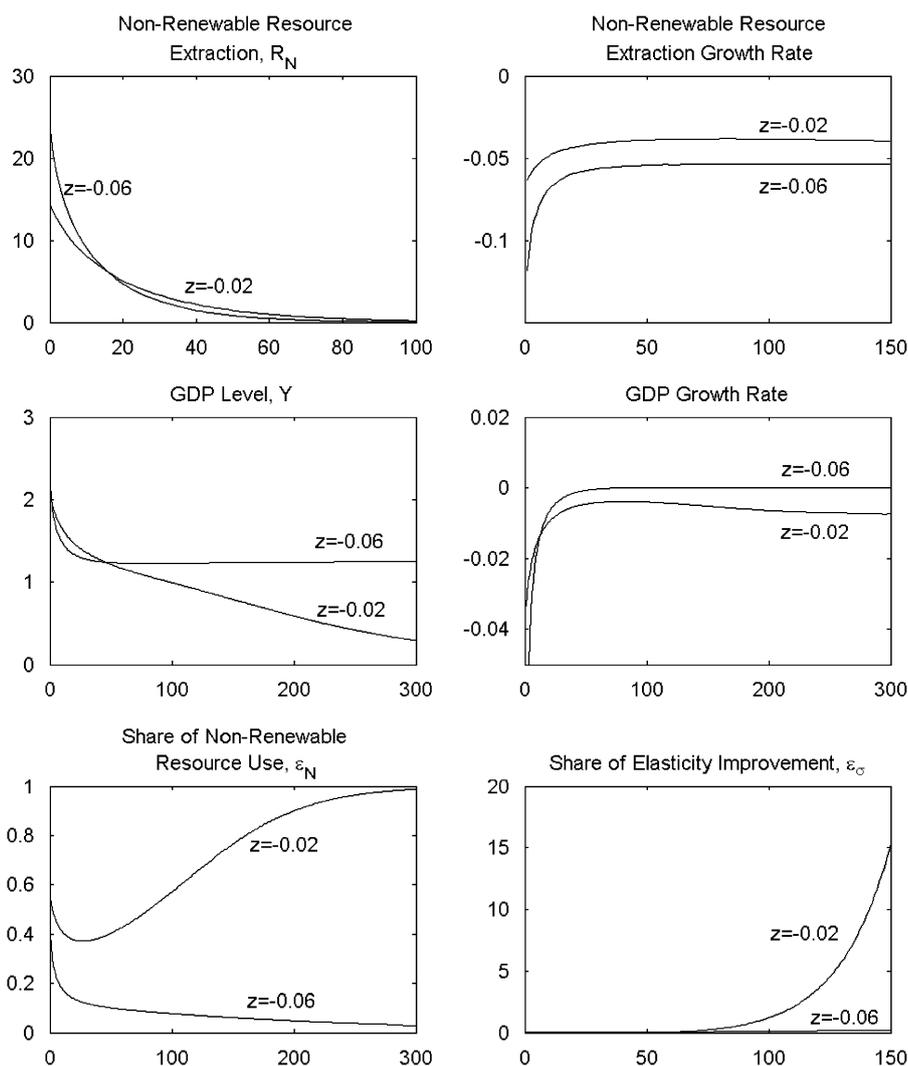


Fig. 6. Biased technical change case,  $s = 0$ ,  $g = 0$ . Dependence on the magnitude of the bias in technical change,  $z$ .

and finally, a better substitutability between resource inputs allows for much quicker reactions to potential price changes.<sup>19</sup> Higher substitutability implies that when prices of non-renewable inputs rise as drastically as oil prices did during the period 2005–2008, the use of renewable resources ought to increase substantially, substituting out non-renewable resources, lowering the demand for them, and consequently smoothing the price shock.

**Corollary 2.** Our analysis suggests that the effect of factor-augmenting technical change depends crucially on the factor towards which it is directed. Two cases emerge here: (i) if the two types of resources are gross substitutes, then the effect is positive if it is directed to the resource which is used in relatively larger amounts for production, otherwise it is negative; (ii) if the two types of resources are gross complements, then the effect is positive if it is directed to the resource which is used in relatively smaller amounts. In terms of policy suggestions, we find that new technologies increasing the unit productivity should currently be directed to renewable resources

(since the EoS is arguably low, and non-renewable resources are for sure used in relative abundance). However, if renewable resources become more heavily used in production than non-renewable ones in the future, then productivity-augmenting technical change should be re-directed to non-renewable resources. A similar shift in emphasis should be made if the EoS would appear to exceed unity. Finally, a shift back to renewable resources would be desirable in the case EoS exceeds unity and renewable resources are relatively more heavily used in production.

These results need to be verified within an endogenous technical change framework, but one can be nevertheless confident that our assertions would hold in an endogenous technical change framework as well: they have been obtained from the basic properties of the production function itself.

One crucial variable here (regarding the least favourable case of  $\sigma < 1$  and no technical change able to improve it) is the expected speed of biased technical change. Our results show that if the speed of biased technical change towards the scarce non-renewable resource can outweigh the reductions in its flows, then production can be non-declining in the long run even though these resources will be ultimately depleted. The pace of reductions in the flows of non-renewable resources depends on the

<sup>19</sup> This result does not follow from our model directly but it is easy to generalise it such that the quoted statements are derived formally.

time preference (too much impatience leads to too fast reductions), on inequality aversion (if we do not want to spread consumption over time we will use up non-renewable resources too quickly), and on the substitutability between the resource inputs. The last result is also crucial for the type of technical change on which one should focus. If resource inputs are near perfect complements or it is hardly possible to improve the substitutability between these inputs, then it would be useful to invest in improving the unit productivities, where the improvements should be directed to the resource which is used in relatively larger amounts.

**Corollary 3.** *We see that the renewable resource works as a potential (or conditional) backstop technology in this model. It is a backstop technology (cf. Nordhaus, 1973) because it is a technology which allows to produce positive output forever without the need to use non-renewable resources as well. But it is only potential, because the economy has first to assure that the EoS between the two resource inputs exceeds one. Of course, this need not be the case: even if we allow for factor-augmenting technical change in both types of resources,  $\sigma$  may well stay below unity. The benchmark case with  $\sigma \leq 1$  and the biased technical change case with  $\sigma < 1$  illustrates the trouble with such a situation.*

Whereas in the benchmark case and in the biased technical change case, the renewable resource works as a conditional backstop technology (the condition being that  $\sigma > 1$ ), an EoS increasing over time in a way which ensures crossing unity gives it the characteristic of a *dynamically emerging* backstop technology. In such case, this backstop technology is known from the start, the “application” of this technology is certain, but the timing depends crucially on the growth rate of EoS.

We also find that once the conditions for the usage of renewable resources as a backstop technology are satisfied, it becomes profitable to deplete the non-renewable resource in finite time.

**Corollary 4.** *Our analysis suggests an important final corollary: sustainability of income depends crucially on the type of technical change. Of course, this opens further vital questions: can we, in the near future, improve the substitutability between non-renewable and renewable resources sufficiently in order to make non-renewable inputs inessential? Do we expect that biased technical change directed towards the more heavily used resource will be fast enough to compensate for the declining flows of non-renewable inputs? It is clear that small improvements in the EoS may have large impacts on total output produced even though both inputs are essential. It might therefore be the case that we first would wish to increase the EoS even though we believe that we cannot make non-renewable inputs inessential, and then secondly try to compensate the decreasing non-renewable input by improvements in the unit efficiencies. Only a full model which incorporates both types of technical change in an endogenous way will be able to give definite answers. This remains on our research agenda.*

## Conclusion

This article has investigated two non-standard ways of looking at technical change: technical change in the elasticity of substitution (EoS) as well as in partial elasticities of the resource inputs (in the production function). We have used a simple economic growth model to compare the long- and short-run implications of these two types of technical change with the ones obtained within a well established benchmark model.

Continued substitutability improvements have two effects. One, they improve upon the technical efficiency of the overall

resource bundle. Two, they help render the non-renewable resource inessential for production from some time on. Our analysis assumed that the policy maker would take the changes in the EoS into account when choosing the optimal time path of non-renewable resource extraction. Basing on this assumption, we have established two findings: (i) the primary short-run effect of increasing the EoS is the increase in efficiency, (ii) in the long run, the EoS acts mostly as a “dummy” for essentiality.

Biased technical change means that both resource inputs are subject to technical change, but that the productivity of one of the resources improves quicker than the other resource. We find that technical change is most useful when it is directed to the resource which is currently more important for production, and in the long run, it is especially useful if it is quick enough to compensate for reductions in the flows of extracted non-renewable resources. Quantitative changes in the speed of biased technical change can bring about qualitative differences in the long-run outcome of the model.

The strongest message of this article is however that the EoS plays a more important role for the long-run dynamics even than the *growth rate* of the distribution parameter between the resource inputs. In an extreme case, a one-shot improvement in the substitutability can be more beneficial for the economy than perpetual factor-augmenting technical change.

In the light of these findings, consecutive research should address the following points. Firstly, exogenous technical change in the substitutability should be endogenised. It is certainly one of the most important parameters, if not the most important, in models with non-renewable resources. This is by no means a simple task and valuable results might only be attainable for special cases. Secondly, one should attempt to compare the outcome of a model with endogenous technical change in the EoS with the outcomes of models featuring endogenous biased technical change in both resource inputs, already present in literature. Such results would help in deepening our understanding as to which kind of technical change should be promoted by policy makers. Clearly, it is difficult to use the standard balanced growth approach to capture an optimal shift in research towards one of the types of resources. Therefore, the question remains, how can one model the possibility of an optimal shift of R&D effort from one sector to the other.

Thirdly and finally, our analysis has (partly) been conducted in response to the bleak outlook painted by Cleveland and Ruth (1997), who suggest that the “traditional” types of technical change do not seem to be fast enough. Knowing the way in which technical change in the EoS as well as in the distribution parameters updates the existing results, it now remains to be asked whether empirical evidence still gives the same bleak outlook as before.

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## Appendix

### Optimisation of the Ramsey–Hotelling model

We can write the Hamiltonian as follows (omitting time subscripts):

$$H(R_N, S_N, \lambda) = U\left(A[\psi R_N^\theta + (1 - \psi)R_R^\theta]^{\beta/\theta}\right)e^{-\rho t} - \lambda R_N. \quad (9)$$

The Pontryagin maximum conditions are

$$\frac{\partial H}{\partial R_N} = 0 \Rightarrow \lambda = \frac{\partial U}{\partial R_N} e^{-\rho t}, \quad (10)$$

$$\frac{\partial H}{\partial S_N} = -\dot{\lambda} \Rightarrow -\dot{\lambda} = 0. \quad (11)$$

Differentiating Eq. (10) with respect to time and substituting into Eq. (11) gives

$$\hat{Y} = \frac{\hat{F}_N - \rho}{\gamma}, \quad (12)$$

which is Eq. (7) in the main text. The marginal product of the non-renewable resource  $R_N$  is given by  $F_N = A\beta\psi R_N^{\theta-1}R^{\beta-\theta}$ . Its growth rate is given by

$$\hat{F}_N = g + z + (\theta - 1)\hat{R}_N + (\beta - \theta)(\varepsilon_N \hat{R}_N + \varepsilon_\psi z + \varepsilon_\sigma s) + \frac{1}{\sigma}[\log(R_N) - \log(R)]s. \quad (13)$$

As the growth rate of income is  $\hat{Y} = g + \beta(\varepsilon_N \hat{R}_N + \varepsilon_\psi z + \varepsilon_\sigma s)$ , we can substitute these two growth terms into Eq. (12) to get

$$(1 - \gamma)g + [1 + (\beta - \theta)\varepsilon_\psi - \gamma\beta\varepsilon_\psi]z + [(\beta - \theta)\varepsilon_\sigma - \gamma\beta\varepsilon_\sigma]s - \rho + \frac{1}{\sigma}[\log(R_N) - \log(R)]s = [\gamma\beta\varepsilon_N + 1 - \theta - (\beta - \theta)\varepsilon_N]\hat{R}_N.$$

Solving for  $\hat{R}_N$  gives us the optimal growth rate of the non-renewable resource extraction, Eq. (8).

### Proof that the denominator of $\hat{R}_N$ is always positive

The denominator of  $\hat{R}_N$  is given by  $(1 - \theta) - ((1 - \gamma)\beta - \theta)\varepsilon_N$ . Rewriting this gives  $(1 - \theta) - (1 - \gamma)\beta\varepsilon_N + \theta\varepsilon_N + \varepsilon_N - \varepsilon_N = (1 - \varepsilon_N)(1 - \theta) - [(1 - \gamma)\beta - 1]\varepsilon_N$ . As we know that  $\varepsilon_N \in [0, 1]$  and  $\theta \in (-\infty, 1]$ , we also know that the first term is greater or equal to zero. As  $\gamma \in (0, \infty)$ ,  $0 < \beta < 1$ , then  $(1 - \gamma)\beta - 1 < 0$ , so the denominator turns out to be a sum of two non-negative expressions,

one of them being strictly positive. This implies that  $(1 - \theta) - ((1 - \gamma)\beta - \theta)\varepsilon_N > 0$ .

## References

- Acemoglu, D., 2003. Labor- and capital-augmenting technical change. *J. Eur. Econ. Assoc.* 1, 1–37.
- Amigues, J.-P., Moreaux, M., Ricci, F., 2006. Overcoming the natural resource constraint through dedicated R&D effort with heterogenous labor supply. Mimeo, Université Toulouse 1 & LEERNA.
- André, F., Cerdá, E., 2005. On natural resource substitution. *Resour. Policy* 30 (4), 233–246.
- Arrow, K., Chenery, H., Minhas, B., Solow, R., 1961. Capital–labor substitution and economic efficiency. *Rev. Econ. Stat.* 43, 225–250.
- Bretschger, L., 2005. Economics of technological change and the natural environment: how effective are innovations as a remedy for resource scarcity? *Ecol. Econ.* 54 (2–3), 148–163.
- Cleveland, C., Ruth, M., 1997. When, where, and by how much do biophysical limits constrain the economic process? The contribution of Nicholas Georgescu-Roegen to ecological economics. *Ecol. Econ.* 22, 203–223.
- Dasgupta, P., Heal, G., 1974. The optimal depletion of exhaustible resources. *Rev. Econ. Stud.* 41, 3–28 (symposium on the economics of exhaustible resources).
- de La Grandville, O., 1989. In quest of the Slutsky diamond. *Am. Econ. Rev.* 79, 468–481.
- Grimaud, A., Rouge, L., 2005. Polluting non-renewable resources, innovation and growth: welfare and environmental policy. *Resour. Energy Econ.* 27 (2), 109–129.
- Growiec, J., Schumacher, I., 2006. On technical change in the elasticities of resource inputs, CORE Discussion Paper 2006/63.
- Hicks, J., 1932. *The Theory of Wages*, second ed. Macmillan, London, (1963).
- Klump, R., Preissler, H., 2000. CES production functions and economic growth. *Scand. J. Econ.* 102, 41–56.
- Miyagiwa, K., Papageorgiou, C., 2007. Endogenous aggregate elasticity of substitution. *J. Econ. Dyn. Control* 31 (9), 2899–2919.
- Nordhaus, W., 1973. The Allocation of Energy Reserves, *Brookings Papers* 3, pp. 529–570.
- Petith, H., 2001. Georgescu-Roegen Versus Solow/Stiglitz and the Convergence to the Cobb–Douglas. UFAE and IAE Working Paper 489.01, Universitat Autònoma de Barcelona.
- Romer, P., 1990. Endogenous technological change. *J. Political Econ.* 98, 71–102.
- Scholz, C., Ziemes, G., 1999. Exhaustible resources, monopolistic competition, and endogenous growth. *Environ. Resour. Econ.* 13, 169–185.
- Schou, P., 2000. Polluting non-renewable resources and growth. *Environ. Resour. Econ.* 16, 211–227.
- Solow, R., 1956. A contribution to the theory of economic growth. *Q. J. Econ.* 70, 65–94.
- Solow, R., 1974. Intergenerational equity and exhaustible resources. *Rev. Econ. Stud.* 41, 29–45 (symposium on the economics of exhaustible resources).
- Stiglitz, J., 1974. Growth with exhaustible resources: efficient and optimal growth paths. *Rev. Econ. Stud.* 41, 123–138 (symposium on the economics of exhaustible resources).
- Tahvonen, O., Salo, S., 2001. Economic growth and transitions between renewable and non-renewable energy resources. *Eur. Econ. Rev.* 45, 1379–1398.
- Yuhn, K.-H., 1991. Economic growth, technical change biases, and the elasticity of substitution: a test of the de La Grandville hypothesis. *Rev. Econ. Stat.* 73 (2), 340–346.