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WHEN SHOULD WE STOP EXTRACTING NONRENEWABLE RESOURCES?

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This article analyzes an economy where both nonrenewable resources and a costly energy resource are essential inputs in production. The extraction of the nonrenewable resources leads to emissions that increase the probability of a catastrophe. We find that, in contrast to the constant-probability case, the endogenous probability of a catastrophe implies that some nonrenewable resources might optimally be left in the ground. The larger the effect of the fossil energy use on the probability of a catastrophe, the fewer nonrenewable resources should be extracted and the earlier should be the switch to the renewable substitute. The richer a country, the earlier it should shift to the energy substitute. In the trade-off between higher consumption and a higher probability of catastrophe, even small probability changes are likely to be more important for the planner than higher consumption.

Keywords: Nonrenewable Resources, Renewable Resources, Energy, Catastrophes

1. INTRODUCTION

In this article we take a step toward answering the following question: Given that the increasing use of polluting nonrenewable resources may lead to exceptional disruptions in the global climate, what does this imply for the substitution of a more costly but nonpolluting renewable energy source? Specifically, what determines whether we should fully deplete the stock of nonrenewable resources (such as oil, gas, and coal), or stop using the nonrenewable resource before depletion and switch the energy substitute? Furthermore, what factors influence the timing of the switch? To answer these questions, we suggest taking a combined look at two problems that lie at the heart of modern environmental economics: energy security and the costs of climate change from fossil energy use.

With respect to energy security, we see that fossil fuels represent nearly 80% of world primary energy consumption [IEA (2004)], as it is still cheaper to use nonrenewable energy than its renewable counterpart.[1] Because these

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nonrenewable resources are only available in a limited quantity, we are likely to see their depletion before the twenty-second century [Shafiee and Topal (2009), Schmalensee et al. (1998)]. It is, therefore, necessary to study when it is optimal to switch the more costly renewable substitute.

In addition, the use of fossil fuels frees sufficiently large amounts of CO_2 to change the global climate through the greenhouse gas effect. The more fossil fuels we use the larger the change in global temperatures and the more uncertainties we will face on our road to a sustainable life on planet earth. One of the most substantial uncertainties comes from abrupt climate change. Abrupt climate change means that the climate may transform in such a way as to completely alter the face of the earth. For example, abrupt climate change may come from an interruption of the ocean's thermohaline circulation (part of it being the Gulf Stream). Possible future effects of a change in the ocean's thermohaline circulation are the collapse of the global plankton stocks to less than half their current biomass, with drastic consequences for marine ecosystems [Schmittner (2005)]; sea-level rises [Levermann et al. (2005)]; changes in rainfall patterns such as monsoon failures or droughts in Asia and the Sahel region; and reductions in the carbon dioxide uptake of the ocean and thus positive feedback on global warming [Schiermeier (2006)]. Climate change may also substantially change the cryosphere, the parts of the planet that consist of ice [Pearce (2005)]. Apart from the permafrost's interaction with the climate system, one worry is the potentially huge additional amount of greenhouse gases, particularly methane, that may be set free once the permafrost vanishes. At the end of the Permian era, 250 million years ago, climate change led to the freeing of methane, which is believed to have resulted in the mass extinction of around 70% to 90% of life on the planet [Kennett et al. (2003)]. Another potential issue is the melting of Greenland's ice caps, which could lead to sea-level rises of around six meters.

The current "solution" to both problems, energy security and the impact of fossil energy on climate change, is substituting the virtually CO_2-free alternative energy sources (biomass, geothermal, wind, solar, and marine energy) on a global scale. But in doing so, we have to trade off our prosperity and economic growth, both of which currently so uniquely rely on the cheap but limited and exhaustible resources, with the uncertainties of abrupt climate change from an increasing use of these fossil fuels and the effect of higher energy costs once we substitute the more costly alternative energies. The problem that we are going to analyze here, in other words, is the trade-off between the level of risk that we are willing to accept and the change in current consumption levels once we switch from the use of fossil energy the expensive energy substitutes.[2]

To study these trade-offs, an analytical approach requires the following components. First, we need a framework where a planner optimally chooses between the use of polluting nonrenewable resources and a costly energy substitute. We need to be able to deal with corner solutions and the nonrenewable character of the polluting resource. For this, we borrow the basic models from Dasgupta and Heal (1974), Heal (1976), Tahvonen and Salo (2001), and Jouvet and Schumacher

(2009). In those models, a nonrenewable resource and a costly renewable one are perfect substitutes in the production of capital. In that case the authors find that the nonrenewable resource should be fully depleted, because it only provides a benefit to society.

This result does not need to prevail if we consider that the nonrenewable resource is polluting and therefore bears an effect on the probability of catastrophes. We therefore integrate into the model of Jouvet and Schumacher (2009) the studies of Tsur and Zemel, who extensively work on endogenous catastrophes [see Tsur and Zemel (1996, 2008)]. Tsur and Zemel (1996) assume that the uncertainty derives from ignorance of the exact pollution level that leads to a catastrophe. In our case, we assume, in line with Tsur and Zemel (2008), that the uncertainty relate to the time at which a catastrophe might occur. Tsur and Zemel (2008) study, in a framework similar to ours, the effect and size of an optimal Pigouvian tax. Due to the modeling assumptions in Jouvet and Schumacher (2009), we are—with respect to our specific questions—able to obtain further analytical results than Zemel and Tsur (2008). The contribution of this article is to advance our understanding of the factors that influence the optimal switching time between resource inputs, as well as factors that impact the size of the nonrenewable resource stock left in the ground. Because the first stage of the control problem is similar to the Dasgupta and Heal (1974) model, we are unable to fully solve that part for the dynamics of this model. We therefore complement our analysis with numerical simulations.

Our main results are as follows. In the case where the planner does not consider the effect of the nonrenewable resource extraction on the probability of a catastrophe, the nonrenewable resource will always be fully depleted. If the planner assumes an effect of fossil fuel use on the probability of a catastrophe, then some nonrenewable resources might optimally be left in the ground. The larger the effect of the fossil fuel use on the probability of a catastrophe, the less nonrenewable resource will be extracted and the earlier should be the switch to the renewable resource. Richer countries should shift earlier to the energy substitute. Furthermore, even though the sensitivity of the probability of catastrophes varies substantially for some simulations, the planner optimally choses an extraction path such that the differences in the final discount factor are minimal. The effects on consumption and capital are, however, substantial. This suggests that a planner, in the trade-off between a higher discount rate and consumption, is likely to prefer lower consumption to a higher probability of catastrophe.

The article is structured as follows. Section 2 introduces the model and solves for the main results. In Section 3 we study further questions numerically. Section 4 concludes.

2. THE MODEL

We base our analysis on a representative-agent approach where the agent maximizes his indefinitely long stream of utility $u(C)$, which is derived from consumption C_t. Capital stock K_t is reduced by consumption but can be increased via production $F(K_t, R_t + M_t)$, and production is generated from using capital and energy, where both fossil energy R_t and alternative energy M_t may be utilized. Fossil energy comes at a negligible cost from a limited stock S_t,[3] whereas the production of alternative energy requires the use of capital at a constant per unit cost $\gamma > 0$. Assuming constant marginal costs in the production of alternative energy sources may not be very realistic, but it does not affect the main results of our analysis [see Chakravorty et al. (1997) and Zemel and Tsur (2008)]. The use of fossil energy increases the probability of abrupt climate changes, and thus the lower the stock of fossil fuels (for a given initial stock S_0), the higher the probability that a catastrophe may occur. In other words, the stock of nonrenewable resources S_0 includes "sleeping" carbon, which, if set free, induces an increase in the probability of a catastrophe. Therefore, the more is extracted, the higher the probability of a catastrophe, implying an inverse relationship between the probability of a catastrophe and the amount S_t left in the ground. We therefore write $\beta(S_t)$ as the probability of a catastrophe given there is an amount S_t left in the ground. We are uncertain about when the catastrophe is going to occur. Our objective function is thus

$$E_{t_0}\left\{\int_0^T u(C_t)e^{-rt}dt\right\},$$

where E represents the expectation operator at time $t = 0$. Here we assume that the catastrophe at date T leads to complete destruction or to such a disruption in the system that we cannot plan ahead for the time afterward. We shall rewrite the objective function by making use of a Poisson-type probability. The probability of a catastrophe having occurred at time t is equal to the c.d.f. $G_t = P(\tau \leq t) = 1 - \Gamma_t$, where Γ_t is the survival function, which is the c.d.f. of no catastrophe having occurred until time t. Thus, $\Gamma_t = \exp\{-\int_0^t \beta(S_\tau)d\tau\}$, where $\beta(S_\tau) = \lim_{\epsilon\downarrow 0}([\Gamma_\tau - \Gamma_{\tau+\epsilon}]/\epsilon\Gamma_\tau)$. We can then rewrite the expectation as

$$E\left\{\int_0^T u(C_t)e^{-rt}dt\right\} = \int_0^\infty u(C_t)e^{-rt}e^{-\int_0^t \beta(S_\tau)d\tau}dt.$$

For simplicity, we shall define $r + \beta(S) = \rho(S)$. The optimal control problem of the representative agent is then fully described by the following set of equations:

$$\max_{\{C_t, R_t, M_t\}} \int_{t=0}^\infty u(C_t)e^{-\Delta_t}dt \quad \text{subject to} \tag{1}$$

$$\begin{cases} \dot{K}_t = F(K_t, R_t + M_t) - C_t - \gamma M_t, & \forall t \\ \dot{\Delta}_t = \rho(S_t), & \forall t \\ \dot{S}_t = -R_t, & \forall t \\ R_t, M_t, K_t, C_t, S_t \geq 0, & \forall t, \\ \text{with } K_0,\ S_0 \text{ given.} & \end{cases}$$

The particular assumptions are

ASSUMPTION 1. *We assume that* $F : \mathbf{R}^3_+ \to \mathbf{R}_+$, $F_K > 0$, $F_R > 0$, *and second derivatives negative, cross derivatives positive. Furthermore, function* $F(K, R + M)$ *has constant returns to scale in* $R + M$ *and* K.

The assumption of perfect substitution between nonrenewable resources R and their substitute M is borrowed from Tahvonen and Salo (2001). Similarly, the assumption of constant returns to scale (CRTS) is widely used in nonrenewable resource models [see, e.g., Dasgupta and Heal (1974), Heal (1976), and Tahvonen and Salo (2001)]. It is clearly a simplifying assumption that allows us to specifically focus on the ratio of K to $R + M$ as well as on a balanced growth path.[4]

ASSUMPTION 2. *The utility function* $u : \mathbf{R}_+ \to \mathbf{R}_+$ *is at least twice continuously differentiable and has the standard properties of* $u'(C) > 0$, $u''(C) < 0$, $\forall C$. *We assume that* $u'(0) = \infty$.

The assumption $u'(0) = \infty$ allows us to concentrate on interior solutions only. It corresponds to the assumption that at least a minimum amount of consumption is required for the continuation of the generations.

ASSUMPTION 3. *We assume that the at least twice differentiable discount rate* $\rho(S) : \mathbf{R}_+ \to \mathbf{R}_{++}$ *has the properties* $\lim_{S \to S_0} \rho(S) = r > 0$, $\rho'(S) < 0$, $\rho''(S) \geq 0\ \forall S$.

We now study the control problem. We define the optimization problem by introducing the discount factor as another constraint, which allows the Hamiltonian to be independent of time and greatly simplifies the analysis.[5] We also neglect time subscripts whenever possible.

The Hamiltonian of this problem is written

$$\mathcal{H} = u(C)e^{-\Delta} + \lambda \dot{K} + \mu \dot{S} - \phi \dot{\Delta}, \tag{2}$$

and the Langrangian with the nonnegativity constraints can be written, with slight abuse of notation, as

$$\mathcal{L} = \mathcal{H} + \omega_R R + \omega_M M. \tag{3}$$

First-order conditions then give

$$u'(C)e^{-\Delta} = \lambda, \tag{4}$$

$$-\lambda F_K = \dot{\lambda}, \tag{5}$$

$$\lambda(F_R - \gamma) + \omega_M = 0, \quad \omega_M M = 0, \tag{6}$$

$$\lambda F_R - \mu + \omega_R = 0, \quad \omega_R R = 0, \tag{7}$$

$$\phi \rho'(S) = \dot{\mu}, \tag{8}$$

$$-u(C)e^{-\Delta} = \dot{\phi}. \tag{9}$$

The transversality condition is

$$\lim_{t\to\infty} \mathcal{H} = 0,$$

as derived by Michel (1982). We also have $\lim_{t\to\infty} \mu S = 0$ and $\lim_{t\to\infty} \lambda K = 0$.

2.1. Resource Use in the Transition Period

We wish to know how we are going to use the fossil energy and the alternative energy in the transition period. The next results follow from the analysis of Jouvet and Schumacher (2009). We assume, for now, that $\rho'(S) = 0$. Making use of the complementary slackness conditions, we see that $R > 0$ and $M = 0$ implies $\lambda F_R = \mu$ and $F_R < \gamma$. Thus, if capital and the nonrenewable resource exist in sufficient amounts such that $F_R < \gamma$, and if $\lambda\gamma > \mu$, then $R > 0$ and $M = 0$. Assuming that $R > 0$ and $M > 0$ implies that $\lambda\gamma = \mu$, which is possible only at one point in time, because $\hat{\lambda} < 0$ and $\hat{\mu} = 0$. Assume $R = 0$ and $M > 0$; then $F_R = \gamma$ and $\lambda\gamma < \mu$. Conclusively, for $\rho'(S) = 0$, the fossil energy and the alternative energy may only be used simultaneously at one instant in time. We should then have depleted the nonrenewable resource at the point in time when it became efficient to use the costly substitute.[6] Allow now for $\rho'(S) < 0$. In this case the previous analysis still holds, but because $\hat{\mu} \neq 0$, we may have intervals where $R > 0$ and $M > 0$, which then requires $\mu = \gamma\lambda$. The previous result, $R > 0$ and $M = 0$ if $\lambda F_R = \mu$ and $F_R < \gamma$, still holds. For example, if the change in the probability of a catastrophe given a change in the amount of CO_2 in the atmosphere is very small and future consumption is expected to be low, then changes in μ are likely to be small.

If we therefore start with the realistic assumption that $M = 0$ and $R > 0$, then $\lambda F_R = \mu$ and we derive a new Hotelling rule:

$$\hat{F}_R = F_K + \frac{\rho'(S)}{\rho(S) F_R}\left[\frac{u(C)}{u'(C)} + \dot{K} - F_R R\right], \tag{10}$$

or more simply,

$$\hat{F}_R = F_K + \frac{\rho'(S)\phi}{u'(C)e^{-\Delta} F_R}.$$

The intuition for the last condition is as follows: Hotelling's original rule suggests that the nonrenewable resource should be extracted at an amount that equalizes the percentage change in its price with the additional value it helps to create in production, as this maximizes the present value of the resource. In our case, the policy maker needs to take into account that this extraction leads to more carbon in the atmosphere, which then increases the probability of a catastrophe. Therefore, the policy maker will reevaluate the price changes of the resource by taking this additional cost of extraction into account. However, if consumption is very low or the marginal product of the resource very high, then the policy maker is likely to put less emphasis on the future possibility of a catastrophe.

Writing $x = K/R$ and applying Euler's theorem for homogeneous functions of degree 1, we get $F(K, R) = Rf(x)$, and thus we obtain

$$\frac{\dot{x}}{x} = \frac{\eta f(x)}{x} - \frac{\rho'(S)\phi}{u'(C)e^{-\Delta} f''(x)x}, \tag{11}$$

where

$$\eta = -\frac{f'(x)[f(x) - xf'(x)]}{xf(x)f''(x)} \in [0, \infty]$$

is the elasticity of substitution between capital and energy. Thus, $\dot{x}/x > 0$ if

$$\eta f(x)f''(x) < \frac{\rho'(S)\phi}{u'(C)e^{-\Delta}}.$$

On the left-hand side we have the variables describing the technological situation, whereas on the right-hand side we find the ones describing preferences. From this we can deduce that $\dot{x}/x > 0$ if substitution between resources is easy (large η), or if the capital–resource ratio is very low. Furthermore, if current consumption is low, then we should not care too much about future consumption. Conversely, if the probability of a catastrophe increases drastically with more carbon in the atmosphere, then $\dot{x}/x < 0$.

The growth rates of the shadow prices are given by

$$\frac{\dot{\lambda}}{\lambda} = -F_K < 0, \tag{12}$$

$$\left.\frac{\dot{\mu}}{\mu}\right|_{R>0} = \frac{\rho'(S)}{\rho(S)F_R}\left[\frac{u(C)}{u'(C)} + \dot{K} - F_R R\right] < 0, \tag{13}$$

$$\left.\frac{\dot{\mu}}{\mu}\right|_{R=0} = \frac{\rho'(S)}{\rho(S)}\frac{1}{\lambda F_R + \omega_R}\left[u(C)e^{-\Delta} + \lambda\dot{K}\right] < 0. \tag{14}$$

Clearly, both shadow values are nonnegative. If we integrate (9) forward we obtain that ϕ_t gives the prospective value of future utility,

$$\phi_t = \int_t^\infty u(C_\tau)e^{-\Delta_\tau}\, d\tau. \tag{15}$$

Similarly, equation (8) gives the prospective future shadow value of the resource stock,

$$\mu_t = -\int_t^\infty \phi_\tau \rho'(S_\tau)\, d\tau > 0. \tag{16}$$

Using the transversality condition $\lim_{t\to\infty} \mu_t S_t = 0$, we obtain the condition

$$\lim_{t\to\infty} -S_t \int_t^\infty \phi_\tau \rho'(S_\tau)\, d\tau = 0. \tag{17}$$

Conclusively, fossil energy will be fully depleted as long as its value toward utility in the limit is positive. In the case of $\rho'(S) = 0$, we have that $\dot{\mu} = 0$, $\mu > 0$, so that the transversality condition $\lim_{t\to\infty} \mu_t S_t = 0$ requires that $\lim_{t\to\infty} S_t = 0$. Thus, as long as the extraction of the nonrenewable resource does not increase the probability of a catastrophe, the resource stock will be fully depleted. Assume now that $\rho'(S) < 0$. In this case, given that $\hat{\mu}_t \leq 0 \ \forall t \geq 0$, we may have $\lim_{t\to\infty} \mu_t = 0$, implying that $\lim_{t\to\infty} S_t > 0$ may be optimal (because the transversality condition $\lim_{t\to\infty} \mu_t S_t = 0$ can still be satisfied). Thus, the prospect that a lower stock increases the probability of abrupt climate disruptions implies that not all fossil energy should be extracted. The timing here is decisive. The more likely are abrupt climate disruptions for a given change in the fossil fuel stock, the faster should be the move to alternative energy sources. Similarly, the better-off society is expected to be in the fture, the sooner the change should be made.

In summary, the reason that one switches to the costly substitute without having fully depleted the nonrenewable resource is as follows. Imagine that $\rho'(S)$ is very large. Then the shadow value of the nonrenewable resource will be high. In that case, a unit increase in S will imply a substantial increase in overall welfare, because this reduces the probability of a catastrophe substantially. If now the renewable resource is cheap enough and productive enough, then it is more efficient to use the renewable substitute and leave the rest of the nonrenewable resource in the ground to prevent a further increase in the probability of a catastrophe. This is also the decisive difference in results from the analysis in Dasgupta and Heal (1974), where the nonrenewable resource will be either fully depleted at a finite time (in the case of a backstop technology) or used forever and depleted in the limit (if there is no backstop technology).

2.2. At the Switching Time

Assume now we started initially with a situation where $\lambda_t \gamma > \mu_t$, such that $R_t > 0$ and $M_t = 0$. Assume furthermore that there exists a point in time when $\lambda_t \gamma < \mu_t$, such that $R_t = 0$ and $M_t > 0$, implying that we stop using the fossil fuel and switch to the alternative energy source. Let us denote the switching date as $t = T$. At that moment in time we have $\lambda_T \gamma = \mu_T$. We also know that $\mu_T = -\int_T^\infty \phi_\tau \rho'(S_\tau) d\tau > 0$. Given the analysis in Jouvet and Schumacher (2009), we can derive for $t \geq T$ an explicit solution of ϕ_t in the case of a constant–relative risk aversion (CRRA) utility function and a constant–elasticity of substitution (CES) production function. We shall therefore assume from now on the functional forms $u(C) = C^{1-1/\sigma}/(1 - 1/\sigma)$, with $\sigma > 1$, and $F(K, R + M) = A[\alpha K^\theta + (1 - \alpha)(R + M)^\theta]^{\frac{1}{\theta}}$, with $A > 0$, $\alpha(0, 1)$, and $\theta \in (-\infty, 0]$.[7] The optimal path for consumption for $t \geq T$ is then given by

$$C_t = \{\Psi - \sigma[\Phi - \rho(S_T)]\} K_T e^{\sigma(\Phi - \rho(S_T))(t-T)}, \tag{18}$$

with the parameters given by $\Phi = \alpha A[\alpha + (1-\alpha)\psi]^{(1-\theta)/\theta}$, $\Psi = \big(A[\alpha + (1-\alpha)\psi]^{1/\theta} - \gamma\psi^{1/\theta}\big)$, and

$$\psi = \frac{\alpha}{\left[\frac{\gamma}{(1-\alpha)A}\right]^{\frac{\theta}{1-\theta}} - (1-\alpha)}.$$

Interior solutions and existence require that $\gamma < (1-\alpha)^{1/\theta}A$ and $\sigma(\Phi - \rho(S_T)) < \Psi$. Furthermore, the boundedness of the utility functional requires that $(\sigma - 1)(\Phi - \rho(S_T)) - \rho(S_T) < 0$.

The constant parameters Φ and Ψ have a clear interpretation. $\Phi > 0$ is the marginal product of capital, and $\Psi K > 0$ is net production.

LEMMA 1. *The comparative statics of* Φ *and* Ψ *are the same because* $\Phi = \Psi$.

The proof is delegated to the Appendix. Lemma 1 allows us to write the comparative statics of Φ and Ψ with respect to γ as follows:

$$\frac{\partial \Psi}{\partial \gamma} = \frac{\partial \Phi}{\partial \gamma} = -\left[\frac{\gamma}{(1-\alpha)A}\right]^{1/(\theta-1)} \left\{\frac{\alpha}{1-(1-\alpha)\left[\frac{\gamma}{(1-\alpha)A}\right]^{\theta/(\theta-1)}}\right\}^{1/\theta} < 0.$$

This implies that the marginal product of capital is decreasing in the costs of the energy substitute.

We obtain, for $t = T$,

$$u'(C_T)e^{-\Delta_T}\gamma = -\int_T^{\infty} \rho'(S_T) \int_{\tau}^{\infty} u(C_s)e^{-\rho(S_T)(s-T)} ds\, d\tau, \tag{19}$$

and because $\rho'(S)$ is constant for $t \geq T$ given that $R_t = 0$ for $t \geq T$, we can then solve $\lambda_T\gamma = \mu_T$ and obtain

$$\begin{aligned}&\gamma e^{-\Delta_T}\{\Psi - \sigma[\Phi - \rho(S_T)]\}^{-1/\sigma} K_T^{-1/\sigma}\\ &\quad = -\frac{\sigma\rho'(S_T)\{\Psi - \sigma[\Phi - \rho(S_T)]\}^{(\sigma-1)/\sigma} K_T^{(\sigma-1)/\sigma}}{(\sigma-1)\{(\sigma-1)[\Phi - \rho(S_T)] - \rho(S_T)\}^2}.\end{aligned} \tag{20}$$

Equation (20) plays a key role in the following analysis.

The previous analysis allows us to derive the following results. We recall that the following propositions hold under CRRA utility and the CES production function.

PROPOSITION 1. *The larger the change in the effective discount rate for a change in the stock of nonrenewable resources, the more quickly will the energy substitute be used. If the change in the effective discount rate is sufficiently large, the nonrenewable resource will never be used.*

Proof 1. Both parts of the proposition follow trivially by inspection of equation (20). ∎

This result can be understood as follows. Assume, for simplicity, that $\rho'(S) = 0$, which implies that there is no effect of resource depletion on the probability of a catastrophe. From the previous analysis, we then know that the nonrenewable resource will be fully depleted. Now take the other extreme and assume that $\rho'(S)$ takes a large number. Then from the analysis in Section 2.1, we know that the nonrenewable resource should never be used. By continuity of the previous result, we thus know that for intermediate sensitivities of $\rho(S)$ we should have $S_0 > S_\infty > 0$.

PROPOSITION 2. *The lower the level of capital, the later should be the substitution of the renewable substitute.*

The proof is in the Appendix. Intuitively, a higher K_T implies a higher shadow value of the fossil resource, because it increases the marginal product of the nonrenewable resource. On the other hand, a higher K_T reduces the shadow value of capital, because it implies a higher level of consumption at date T.

One has to keep in mind the limitations of using equation (20) and therefore of Proposition 2. Importantly, S_T and K_T are still endogenous functions of the parameters and initial conditions S_0 and K_0.[8] On the other hand, if K_0 is sufficiently large so that equation (20) is at least satisfied (meaning that $\lambda_0\gamma \geq \mu_0$), then we know that the comparative statics applies.

PROPOSITION 3. *If the effective discount rate is sufficiently large, then a higher cost of the energy substitute implies that one should substitute toward the energy substitute later in time, because* $\lambda'(\gamma) > 0$ *and* $\mu'(\gamma) < 0$. *For a low effective discount rate, a higher cost of the energy substitute implies that* $\lambda'(\gamma) < 0$, *and whether the energy substitute is now used earlier or later depends on the relative sensitivity of the shadow values to* γ.

The proof is in the Appendix. Again, we use the following argument here. Assuming that $\lambda_T\gamma = \mu_T$, then we know that for a sufficiently high ρ we have $\lambda'(\gamma) > 0$ and $\mu'(\gamma) < 0$. Conclusively, the higher the costs of the energy substitute, the later it should be used. For very high levels of γ, we can then conclude that the energy substitute may never be used. For example, in the extreme, for $\gamma = \infty$, we can see from equation (6) that $M > 0$ would require $F_R = \infty$. Therefore, there will never be a second stage, $M_t = 0\ \forall t$, and the model will predict a depletion of the nonrenewable resource in line with the Dasgupta and Heal (1974) analysis. Conversely, for $\gamma = 0$, we would directly jump to the second stage of $M > 0$ and never use the nonrenewable resource (assuming that $\rho'(S) > 0$).

3. SIMULATIONS

We now simulate the model to answer the following questions numerically: How does the probability of a catastrophe affect the timing of the transition between nonrenewable and renewable resources? How does it affect the quantity of the

TABLE 1. The parameters in the simulation

Parameter	Value
α	0.33
S_0	50
K_0	2
σ	1/0.95
θ	−4
γ	1.4
A	1

nonrenewable resource extracted? What is the effect of even small changes on the probability on an extreme event?

We simulate the model with GAMS for 100 time periods using the CONOPT3 solver. We use the parameter combinations in Table 1. The elasticity $\alpha = 0.33$ is standard $\theta = -4$ implies that energy is an essential input in production. $\gamma = 1.4$ suggests that it is more costly to convert a unit of capital into energy than to use it in production directly. $S_0 = 50$ and $K_0 = 2$ implies that we start with a relative abundance of fossil energy. We use $\sigma = 1/0.95$, which is an intertemporal elasticity of substitution less than one.[9]

The endogenous discount rate is modeled according to $\rho(S_t) = r + b/(1 + ab \times S_t)$, where $r = 0.01$ is the time preference rate and $b/(1 + ab \times S_t)$ is the probability of the catastrophe. This probability reaches its maximum at b for $S_t = 0$. In the simulations we change b over the range

$$b \in \{2\%, 3\%, 4\%, 5\%, 6\%, 7\%, 8\%, 9\%, 10\%, 15\%, 20\%, 25\%, 30\%, 40\%\}.$$

With b we change ab in order to normalize $b/(1+ab \times S_t)$ at 0.01. Therefore, for the initial value of $S_0 = 50$, any change in b goes along with a change in ab such that $b/(1 + ab \times S_0) = 0.01$. A higher b is then associated with a more negative slope. This implies that for any given S_t, a higher b (and the associated ab) implies a higher probability of catastrophe. The normalization is necessary to compare the different paths fully. In the simulations we are only going to concentrate on the case of an eventual balanced growth path, and we neglect parameter combinations that lead to declining consumption.

We obtain the following results.

RESULT 1. *The larger the probability of an abrupt climate change for a given stock of carbon in the atmosphere, the earlier should the substitute be utilized.*

The simulations in Figure 1 show that the higher the probability of a catastrophe for a given stock of the nonrenewable resource, the earlier one should stop the extraction. This result seems rather intuitive—because the planner does not want to increase the probability of a catastrophe substantially, he or she should stop the extraction early enough to shift toward the energy substitute.

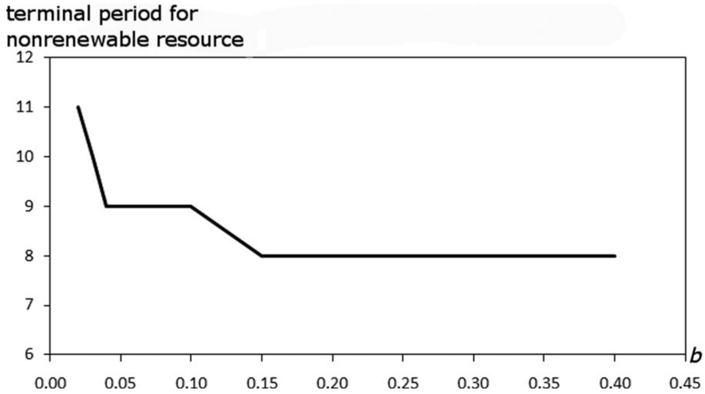

FIGURE 1. Final date of nonrenewable resource use for changing b.

Given the chosen parameters, this shift occurs rather quickly. For lower values of capital or higher costs of the energy substitute, the extraction period will last longer. It is also important to see that for $b \geq 0.15$, the final date of extraction does not change. This is because, given that we normalized the probability of a catastrophe and because extraction is very small, the difference in the final probability of a catastrophe in those cases is small.[10] Thus, the larger the value of b, or the stronger the impact of climate change on the probability of a catastrophe, the faster one should turn to the energy substitute and the more quicker the economy will be on the balanced growth path (and therefore approach its long-run behavior).

RESULT 2. *The larger the probability of an abrupt climate change for a given stock of carbon in the atmosphere, the fewer nonrenewable resources should be extracted.*

As Figure 2 shows, most resources are extracted if the fossil fuel use has little effect on the probability of the catastrophe.

However, we see that not all nonrenewable resources are extracted for even very small effects of the fossil fuel extraction on the probability of the catastrophe. For example, for $b = 2\%$, the stock of the nonrenewable resources is still 38.5 when the planner switches to the renewable resources. This implies that the planner is only willing to accept an increase of less than 0.2% on the discount rate, while still leaving more than 3/4 of the fossil energy in the ground. For values of b that lead to larger changes in the probability of a catastrophe, we even observe that the planner only extracts around 10% of the possible stock of fossil fuels.

This trade-off between accepting higher risks and extracting virtually costless nonrenewable resources is hereby put into an important perspective: A utilitarian

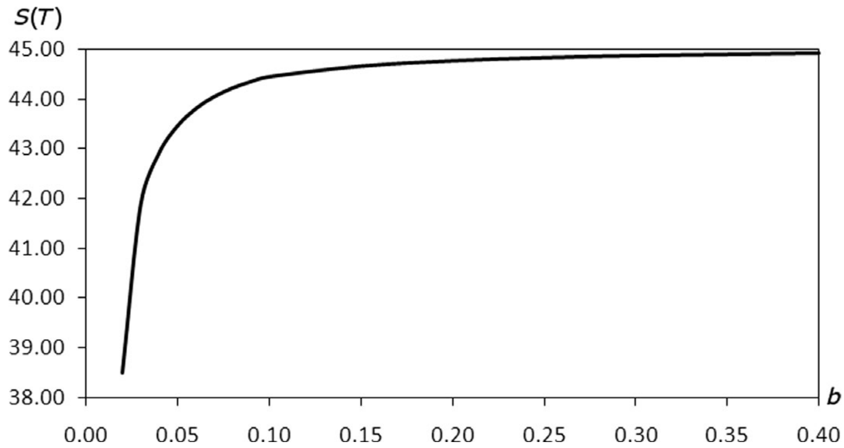

FIGURE 2. Terminal stock of nonrenewable resources for changing *b*.

planner will generally only accept minimal increases in the effective discount rate. This result is shown in Figure 3. Therefore, in the long run, we will see that the planner still optimally leaves some stock of the polluting resource in the ground. This stock is higher the stronger its impact on the probability of a catastrophe.

RESULT 3. *The policy maker prefers to minimize risk at the expense of higher consumption.*

Our last result suggests that the planner carefully trades off the possible higher increases in the effective discount rate with increases in the consumption profile. In the sensitivity analysis in Figure 3, we see that the planner minimizes his impact on the effective discount factor at the expense of higher consumption. In the long run, as shown in Figure 3, the planner may forgo a considerable amount of consumption in order to keep the risk of abrupt climate change low. In the scenarios considered here, the planner might postpone consumption by roughly thirty years in case the maximal probability of a catastrophe (for $S_T = 0$) changed from 2% to 8%. Thus, a smaller probability of a catastrophe implies optimally higher consumption forever. In conclusion, we find that the planner tries to reduce his or her risk exposure even if this costs a substantial amount of his or her consumption.

4. CONCLUSION

In this article, we have analyzed an economy where a nonrenewable resource and a costly energy resource are perfect substitutes in the production of energy. The extraction of the nonrenewable resources implies emissions that lead to increases in the probability of a catastrophe. The costly (but pollution-free) energy substitute

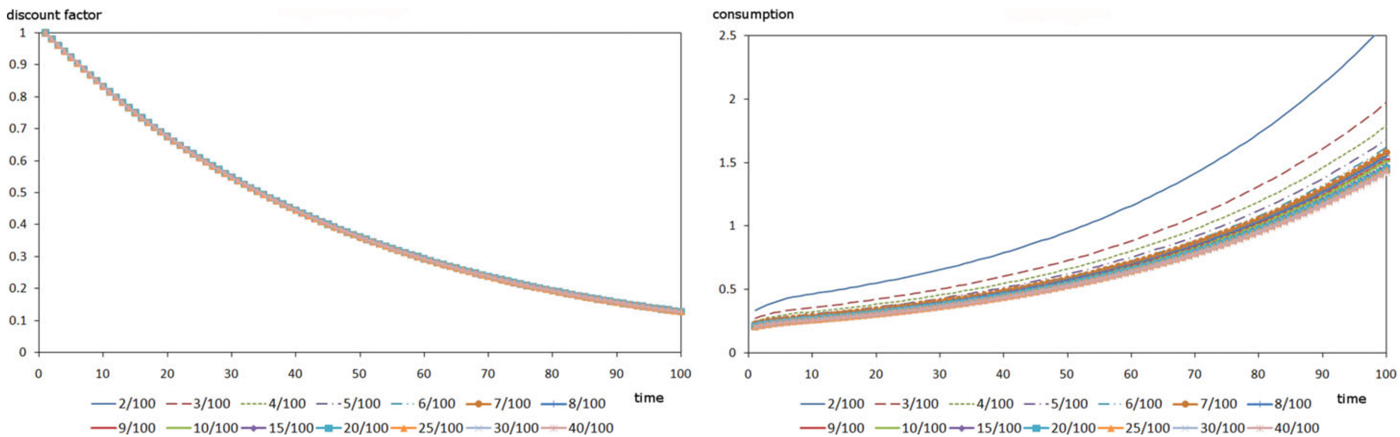

FIGURE 3. Discount factor and consumption profile for different b.

is produced from capital via a CRTS production function. The question addressed is, when should we stop extracting nonrenewable resources? Our main results are that, in the case where the planner does not include the effect of the nonrenewable resource extraction on the probability of a catastrophe, the nonrenewable resource will always be fully depleted. When the planner assumes an effect of fossil fuel use on the probability of a catastrophe, some nonrenewable resources may optimally be left in the ground. The larger the effect of the fossil fuel use on the probability of a catastrophe, the less nonrenewable resource will be extracted and the earlier should be the switch to the renewable resource. Richer countries should shift earlier to the energy substitute.

Furthermore, even though the probability of catastrophes varies substantially for some simulations, the planner optimally chooses his extraction path so that the differences in the final discount factor are minimal. The effects on consumption and capital are, however, substantial. Therefore, we can conclude that in the tradeoff between higher consumption and a higher probability of catastrophe, even small probability changes are likely to be more important to the planner than larger changes in consumption. This is a rather relevant result for the debate on climate change and should provide a somewhat modified perspective on the debate about the role of discounting in the use of fossil energy.

Obviously, any theoretical model and simulation thereof are bound to be limited to some extent. One should, therefore, only view these models as an abstract tool that may be able to replicate certain intrinsic relationships between decision variables while neglecting others that are hoped to be less important. The analysis presented here is certainly subject to those caveats, too, and it can potentially be refined in several ways.

First, we use a rather simple representation of uncertainty as well as of the impact of a catastrophe. In our model, the planner attaches zero welfare to the state after a catastrophe occurred. We assume this to avoid the Dismal Theorem of Weitzman (2009), but also because we simply do not know all the possible implications of a climate catastrophe for our life on earth. A useful extension then would be to model the implications of uncertainty on the postcatastrophe welfare level. Intuitively, one would expect that the higher the probability of the worst state (in our case the zero-welfare state), the closer the results would be to our model.

Second, our model assumes that all types of nonrenewable energy inputs are the same (as are the renewable ones). However, although oil is the most important energy input for transportation, gas is the most important one for heating and coal for electricity production. Though the current coal reserves may be sufficient to keep production going for a considerably further time, the gas reserves may be sufficient to meet our consumption needs only until 2070, whereas oil may only last until 2050. There are, therefore, important complementarities in the production processes that are ignored in this setup and that should ideally be analyzed in a more disaggregated approach.

Finally, the model is, due to its specification, rather difficult to analyze when it comes to switching between nonrenewable and renewable resources. Especially difficult is the theoretical analysis for the case when both resources may be used at the same time. In the simulations we notice that for increasing consumption profiles, the extraction of the nonrenewable resource will stop at a finite point in time, although leaving a positive amount of the nonrenewable resource in the ground. This need not be the case if one also focuses on declining consumption paths. Indeed, in the simulations, a declining path of consumption is generally associated with further use of the nonrenewable resource. Unfortunately, due to its complexity, the model does not allow us to study this case. If one believes that a declining consumption profile (for example, for a high enough cost of the energy substitute or for particularly poor countries) is a realistic possibility, then this deserves another closer look. This will, however, be difficult within the current model.

NOTES

1. The World Energy Outlook suggests that fossil fuels are still expected to account for approximately 85% of energy demand in 2030. Currently, hydropower and "new" renewables (biomass, geothermal, wind, solar, and marine energy) contribute slightly more than 4 % to overall energy consumption, with little change in the relative contribution in recent years.

2. In addition, whereas coal reserves can be found in nearly every region of the planet, oil and gas reserves are mainly located in the Middle East. This energy dependency of certain regions of the world on other regions is bound to lead to international conflicts on as yet unknown scales. Our model can also be used as an ad hoc analysis of the trade-off between the increasing probability of a large-scale and devastating conflict when society tends closer to the depletion of fossil energy and the shift alternative energies.

3. We assume this because from a planner's perspective the costs of fossil energy are negligible. They may only include extraction and transportation costs that are minimal per unit costs. For a consumer in a decentralized setting, however, the costs may be substantially higher and generally include taxes and mark-ups of oligopolistic market structures.

4. If one were to introduce labor, then this would not change the main results with respect to the determinants of the switching time and the factors that influence the amount of nonrenewable resources that are left in the ground. It would, however, prevent a balanced growth path and lead to a steady state instead. This would then, for example, strengthen the conclusions drawn in Result 3.

5. Throughout the article we use $x'(y) = x_y = \partial x/\partial y$, $\dot{x} = \partial x/\partial t$, and $\hat{x} = \partial x/\partial t/x$.

6. The proofs are available from the author.

7. We constrain θ to the range where energy is an essential input in production.

8. Because it is not possible to solve the first stage of the control problem explicitly, it is not possible to derive S_T and K_T fully.

9. Empirical estimates suggest that $\sigma < 1$, which is problematic in models of endogenous discounting. In our case, this would turn μ_t negative, which simply does not make sense. To be sure that our results are not driven by our assumption on σ, we used log utility with initial levels of capital high enough so that consumption always exceeded one. This did not make a difference to the simulation results.

10. Furthermore, because the simulation is in discrete time steps, the graph does not show a smooth curve.

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APPENDIX

Proof 2. Assume $\Phi = \Psi$. This is equivalent to

$$A\alpha \left\{ \alpha - \frac{(1-\alpha)\alpha}{1-\alpha-\left[\frac{\gamma}{A(1-\alpha)}\right]^{\theta/(1-\theta)}} \right\}^{(1-\theta)/\theta}$$

$$= -\gamma \left\{ \frac{-\alpha}{1-\alpha-\left[\frac{\gamma}{A(1-\alpha)}\right]^{\theta/(1-\theta)}} \right\}^{1/\theta} + A \left\{ \alpha - \frac{(1-\alpha)\alpha}{1-\alpha-\left[\frac{\gamma}{A(1-\alpha)}\right]^{\theta/(1-\theta)}} \right\}^{1/\theta}.$$

Simplifying the term inside the largest parentheses gives

$$\alpha - \frac{(1-\alpha)\alpha}{1-\alpha-\left[\frac{\gamma}{A(1-\alpha)}\right]^{\theta/(1-\theta)}} = \frac{-\alpha\left[\frac{\gamma}{A(1-\alpha)}\right]^{\theta/(1-\theta)}}{1-\alpha-\left[\frac{\gamma}{A(1-\alpha)}\right]^{\theta/(1-\theta)}}.$$

Then pulling out the term $[\gamma/A(1-\alpha)]^{\theta/(1-\theta)}$ and dividing by the term in parentheses on the right-hand side leads, after simplification, to no contradiction. ■

Proof 3. Assume $\forall t > T$ such that $\lambda_t \gamma < \mu_t$, implying $R_t = 0$ and $M_t > 0$. Then at $t = T$, we have now that equation (20) holds. By Lemma 1 we can then substitute Φ for Ψ and then define

$$\text{LHS} \equiv \gamma e^{-\Delta_T}[(1-\sigma)\Phi + \sigma\rho(S_T)]^{-1/\sigma} K_T^{-1/\sigma}$$

and

$$\text{RHS} \equiv -\frac{\sigma\rho'(S_T)[(1-\sigma)\Phi + \sigma\rho(S_T)]^{(\sigma-1)/\sigma} K_T^{(\sigma-1)/\sigma}}{(\sigma-1)[(\sigma-1)(\Phi-\rho(S_T)] - \rho(S_T))^2}.$$

We can then easily obtain that $\partial\text{LHS}/\partial K_T < 0$ and $\partial\text{RHS}/\partial K_T > 0$. ■

Proof 4. We will again make use of equation (20). Comparative statics with respect to γ gives

$$\frac{\partial \text{LHS}}{\partial \gamma} = \frac{e^{-\Delta_T} K_T^{-1/\sigma}[\sigma\rho(S_T) + \Phi(\gamma) - \sigma\Phi(\gamma)]^{-(1+\sigma)/\sigma}\left\{\sigma[\sigma\rho(S_T) + \Phi(\gamma) - \sigma\Phi(\gamma)] + \gamma(-1+\sigma)\Phi'(\gamma)\right\}}{\sigma}.$$

We find that $\partial\text{LHS}/\partial\gamma > 0$ if

$$\rho(S_T) > \left[\frac{\gamma}{A(1-\alpha)}\right]^{\theta/(\theta-1)} \left[\frac{\alpha}{1-(1-\alpha)\left(\frac{\gamma}{A-A\alpha}\right)^{\theta/(-1+\theta)}}\right]^{1/\theta}$$
$$\times \frac{A(\sigma-1)}{\sigma^2}\left\{\sigma\left[\frac{\gamma}{A(1-\alpha)}\right]^{\theta/(1-\theta)} - (1-\alpha)(\sigma-1)\right\}.$$

We also obtain

$$\frac{\partial \text{RHS}}{\partial \gamma} = -K^{(-1+\sigma)/\sigma}(1+\sigma)[\sigma\rho(S_T) + \Phi(\gamma) - \sigma\Phi(\gamma)]^{-2-1/\sigma}\rho'[S]\Phi'(\gamma) < 0.$$

Because the left-hand side is increasing for a sufficiently high effective discount rate and the right-hand side is strictly decreasing, Proposition 3 follows immediately. ■