# The endogenous formation of an environmental culture

## revised version

Ingmar Schumacher*

*IPAG Business School*

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IPAG Business School
184 boulevard Saint-Germain
75006 Paris, France
and
Department of Economics
Ecole Polytechnique, Paris, France

Tel.: +352 621264575
E-mail: ingmar.schumacher@ipag.fr

*IPAG Business School, 184 boulevard Saint-Germain, 75006 Paris, France, and Department of Economics, Ecole Polytechnique, Paris, France. Tel.: +352 621264575. E-mail: ingmar.schumacher@ipag.fr

**Abstract**

This article presents a mechanism explaining the surge in environmental culture across the globe. Based upon empirical evidence, we develop an overlapping generations model with environmental quality and endogenous environmental culture. Environmental culture may be costlessly transmitted intergenerationally, or via costly education.

The model predicts that for low wealth levels, society is unable to free resources for environmental culture. In this case, society will only invest in environmental maintenance if environmental quality is sufficiently low. Once society has reached a certain level of economic development, then it may optimally invest a part of its wealth in developing an environmental culture. Environmental culture has not only a positive impact on environmental quality through lower levels of consumption, but it improves the environment through maintenance expenditure for wealth-environment combinations at which, in a restricted model without environmental culture, no maintenance would be undertaken. Environmental culture leads to a society with a higher indirect utility at steady state in comparison to the restricted model.

Our model leads us to the conclusion that, for societies trapped in a situation with low environmental quality, investments in culture may induce positive feedback loops, where more culture raises environmental quality which in turn raises environmental culture. We also discuss how environmental culture may lead to an Environmental Kuznets Curve.

*Keywords:* environmental culture; overlapping generations model; environment; endogenous preferences.
*JEL classification:* Z1; Q56; D90.

# 1 Introduction

The environmental economics literature has extensively studied the role of consumption and abatement decisions for the interplay between economic growth and environmental quality. The focus of this literature has mostly been on the role of technical change, the limits to economic growth (Stokey 1998) imposed by resource constraints (Solow 1974, Dasgupta and Heal 1974), the usefulness of taxes (Jaffe et al. 2003) or educational measures (Prieur and Bréchet 2013). However, in studying these aspects, the literature has barely taken into account cultural aspects (Throsby 2000). Culture plays the important role of shaping the way in which our preferences are directed towards the environment. Consequently, it also affects how we trade off considerations of economic growth and environmental quality. Specifically, Linton (1963, p.466) defines culture as being "the sum total of the knowledge, attitudes and habitual behavior patterns shared and transmitted by the members of a particular society." As a result, culture is not a static concept but develops endogenously. We focus here on the type of culture that affects society's attitude towards the consumption-environment trade-off and dub this particular type of culture the 'environmental culture'. To understand how precisely these changes in attitude affect the consumption-environment trade-off, and the drivers of the social changes, we here study the interplay between environmental culture, economic decisions and the environment.

The first extension to the literature that we develop here is that environmental culture is endogenously determined. The modeling approach shares ideas from Rapoport and Vidal (2007) and John and Pecchenino (1994). In our approach, environmental culture affects how society values consumption relative to environmental quality when old, and it develops costlessly via an intergenerational transmission channel[1], or society may invest into environmental culture via an education channel[2]. The second extension is that the agents' preferences are described by a Leontieff function. It allows to place a stronger emphasis on the fact that, firstly, from an agent's welfare point of view there is only a very limited trade-off between consumption and environment possible. This assumption is akin to the strong sustainability paradigm (see e.g. Daly 1992). Secondly, it suggests that agents place absolute priority on the factor of utility that is needed the most.

The results of this paper are as follows. For low wealth levels, society is unable to free resources for environmental culture. In this case, society will only invest in environmental maintenance if environmental quality is sufficiently low. Once society has reached a certain level of economic development, then it may optimally invest a part of its wealth in developing an environmental culture. When environmental quality and wealth are both sufficiently large, then society may find it optimal to temporarily over-invest (vis-à-vis its steady state level) in

[1] See e.g. Bisin and Verdier (2001, 2005) or Algan and Cahuc (2010) for a general approach, or Graumann and Kruse (1990) and Villacorta et al. (2003) for an environmental setting.

[2] For early empirical evidence see e.g. Van Liere and Dunlap (1980) or, more recently, Franzen and Meyer (2010).

environmental culture. This is optimal until environmental quality is decreased to a level from which onwards it is important for society to also invest in maintenance. In other words, if there is no urgent need for society to improve environmental quality, then society will either invest in environmental culture if it can afford to do so, or not invest in case it is too poor. Then, since investments in environmental culture raise the importance of environmental quality for utility, it becomes also more worthwhile for society to spend money on maintenance due to a positive feedback between culture and the environment. As a result, environmental culture has not only a positive impact on environmental quality through lower levels of consumption, but in addition it improves the environment through maintenance expenditure for wealth-environment combinations at which, in a restricted model without environmental culture, no maintenance would be undertaken.

We, therefore, find that, by raising the importance of environmental quality for utility, environmental culture leads to lower steady state levels of consumption and wealth, but a higher environmental quality. Furthermore, indirect utility at steady state is higher when societies may invest in environmental culture. Thus, for sufficiently rich societies that are trapped in a steady state with low environmental quality, investments in environmental culture may induce positive feedback loops, where a higher appreciation of environmental quality due to environmental culture also induces society to improve environmental quality, which again drives increases in environmental culture. Hence, environmental culture can induce the relationship between economic development and environment as defined in the Environmental Kuznets Curve hypothesis (Grossman and Krueger 1991, Shafik and Bandyopadhyay 1992, Panayotou 1993). Basically, poor countries grow and thereby reduce environmental quality, while richer countries may, for various reasons, be less harmful for environmental quality, thus leading to an inverse u-shape relationship between economic development and pollution. In our case this relationship may occur due to environmental culture, and it can be enforced by technological improvements.

The approach presented in this paper is closely related to the literature on endogenous preferences as well as social norms.[3] However, while a majority of the articles in this line of literature study predetermined preferences, we develop a model of fully endogenous preferences.[4] The article, furthermore, adds to the line of literature on cultural economics (Throsby 2000, Bowles 1998). Specifically, we here combine the major types of capital, namely physical capital, natural capital and cultural capital in one model.[5] Our contribution is to study the endogenous formation of an environmental cultural capital. We think about this

[3]See e.g. Bowles (1998) for a general discussion. Other research includes e.g. endogenous discounting (Becker and Mulligan 1997, Schumacher 2009b), endogenous preferences from religious or group characteristics (Escriche et al. 2004), evolutionary selection or cultural traits (Bisin and Verdier 1998, Bisin and Verdier 2001, Hauk and Saez-Marti 2002).

[4]Predetermined preferences means that, although preferences are endogenously determined in the model, society cannot choose its preferences itself.

[5]For simplicity we neglect human capital, see Becker (2009). This could be a worthwhile extension for future work.

as an intangible form of capital, which defines the common ideas, beliefs, social norms and values of society.

The article is structured as follows. In section 2 we discuss more about the need to model environmental culture and why the choice of lexiographical preferences is useful in this setting. Section 3 introduces the model, section 3.1 solves the temporal equilibrium, and in section 3.2 we solve an explicit version that allows us to derive the global dynamics for the intertemporal equilibrium. In section 4 we discuss the role of environmental culture as a possible mechanism behind the Environmental Kuznets Curve. Finally, section 5 concludes.

# 2 Motivation

In this section we introduce the reader to our concept of an environmental culture, and also discuss more fully our reasons for relying on lexiographical preferences in the model that we present below.

## 2.1 Environmental culture

The anthropologist Linton describes culture as "the sum total of the knowledge, attitudes and habitual behavior patterns shared and transmitted by the members of a particular society." (Linton 1963, p.466). Another known definition is given in Altman and Chemers (1980), who describe culture as consisting of "... beliefs and perceptions, values and norms, customs and behaviors of a group or society." Similar definitions by other anthropologists can be found in Tylor (1958) or Geertz (1957). In economics, culture has been most strongly associated with institutions (e.g. La Porta et al. 1997, Bowles 1998, Platteau 2000), social capital (Putnam and Leonardi 1993) and norms (Bisin and Verdier 2005). We call the specific type of culture that is associated with how mankind treats the planet earth the *environmental culture*. Thus, environmental culture plays a predominant role in shaping how we view, value and subsequently treat trade-offs that affect the environment.

One question is really what shapes an environmental culture. The general approach taken in the literature is that cultural changes are mostly pre-determined.[6] These changes are assumed to be costless. This intergenerational transmission of preferences is studied and analysed in a series of articles by Bisin and Verdier (2001, 2005), who emphasize the

[6] Specifically, Sethi and Somanathan (1996) investigate endogenous social norms in a local common-property resource game. They find that two possible Nash-game equilibria may be stable, one being composed of an individualistic society, the other of a norm-guided society. Brekke et al. (2003) find, in a public good model with social norms directed to effort, that, despite allowing for social norms, this still leads to an under-provision of the public good. Nyborg et al. (2006) add replicator dynamics to the model of Brekke et al. (2003). This results in an equilibrium where either everyone acts according to the green norm, or everyone acts brown. Their social norm rests on the assumption that the moral motivation to act green is large if sufficiently many people act green, whereas the moral motivation is small otherwise. In contrast, Schumacher (2009a) assumes that more people turn environmentalists, or adopt a green culture, when pollution is high, while preferences get directed less towards the environment when pollution is low.

role of cultural dynamics and norms via social interactions across generations. Empirical evidence supporting this intergenerational transmission is, for example, provided in Algan and Cahuc (2010), who show that trust is inherited and thereby affects economic outcomes. Graumann and Kruse (1990) find that attitudes are a social construct, implying that society plays a significant role in shaping preferences. Dalhouse and Frideres (1996) conclude that children tend to adopt the political position of their parents. Villacorta et al. (2003) conclude that children's environmental self-regulation is shaped by their parents' environmental self-regulation. We dub this the intergenerational transmission channel.

In addition to the costless intergenerational transmission of educational culture, there exists ample evidence suggesting that an environmental culture develops in response to educational measures, pressure groups, or simply through spending more time in nature itself. In this respect, it can be shown that countries with higher average educational levels also have a stronger environmental culture.[7] Further empirical evidence supporting this link is available in (e.g. Blomquist and Whitehead 1998, Engel and Pötschke 1998, Danielson et al. 1995, Schumacher 2013b). The relation to social capital has been investigated in Ostrom (1990) and Owen and Videras (2007). Also, Torgler and Garcia-Valiñas (2007) provide an overview of the literature that, among others, investigated the role of education in shaping environmental culture. In contrast to the intergenerational transmission, shaping environmental culture via educational measures tends to be costly. Costs associated with improvements in environmental culture may, among others, arise through teaching environmental values at school, through outdoor learning via e.g. field trips, through information provision (e.g. ecolabels), or social norms that induce inefficiencies which we, for simplicity, denote in monetary terms. The role of education for the shaping of an environmental culture has already been emphasized in a conference on the need for environmental education by the UNESCO/UNEP in 1977, which resulted in the Tbilisi Declaration. In this document, it was suggested that (p. 13-14) "...adopting a holistic approach, rooted in a broad interdisciplinary base, [environmental education] recreates an overall perspective which acknowledges the fact that natural environment and man-made environment are profoundly interdependent." Furthermore, emphasis was placed on the point that environmental education should (p. 14) "... encourage those ethical, economic and esthetic values which, constituting the basis of self-discipline, will further the development of conduct and improvement of the environment."

To summarize, we identified two broad channels that may shape an environmental culture. One channel is a channel of intergenerationally transmitted environmental culture, which we assume is costless. The other channel is an educational channel, which requires a costly investment in order to shape environmental culture.

[7] See Figure 1 in Schumacher (2013a).

### 2.2 Preference structure

In general, economists assume that society can trade off consumption and environmental quality in their utility function along a smooth isoquant. This implies that society may stay on the same utility isoquant if it gives up a little environmental quality to gain a little more of consumption. Nevertheless, this trade-off very often does not, or cannot, happen. This is especially true for societies that are too poor to be able to concern themselves with the environment, or those where environmental quality is so low that it would require immediate attention over any thoughts of wealth accumulation. As a consequence, consumption and environmental quality in the utility can not always be easily substituted (Georgescu-Roegen 1954),[8] and society may wish to allocate resources according to a hierarchy of needs.[9] We shall follow much of the economics literature and model this hierarchy of needs by a Leontieff preference function.

In terms of empirical evidence, Diekmann and Franzen (1999) have shown that poorer societies rank environmental problems lower in comparison to other problems, while they view the severity of these problems similarly to rich societies. Other empirical studies have shown that around a quarter of individuals apply lexicographical preferences when it comes to wildlife preservation (Stevens et al. 1991) or to wetland preservation (Spash 2000). Consequently, societies may rank social decisions according to their needs, and address them mostly consecutively. A poor country may, therefore, be more consumption-oriented during its development stage, while a sufficiently rich country may have enough financial resources left over to treat the more pressing environmental problems. Only those societies that are at the same time sufficiently rich and have a satisfactory level of environmental quality may actually be able to trade off the two in the conventional sense of economic trade-offs.

## 3 The Model

In terms of motivation for the basic modeling structure we follow the convincing approach in John and Pecchenino (1994), with our main changes being the use of the Leontieff utility function and the introduction of environmental culture. Thus, we assume that society's decisions are taken by a representative agent, who may raise taxes to finance environmental maintenance (abatement) or to increase environmental culture. This agent thus optimally allocates income towards maintaining environmental quality, improving environmental cul-

[8]This assumption on the utility function is also an intermediate position to the debate on strong versus weak sustainability. Brekke (1997) suggests that "[a] development is ... said to be weakly sustainable if the development is non-diminishing from generation to generation." In contrast, strong sustainability requires that both human and natural capital are kept intact (Brekke 1997, Daly 1992). While strong sustainability is a requirement that is nearly impossible to be met for a developing society, weak sustainability may induce unsustainable levels of environmental quality. Instead, lexiographical preferences place attention on the input to utility that society ranks as being most crucial.

[9]This has been suggested by Maslow's hierarchy of needs.

ture, or increasing savings for consumption when old. We denote time by subscript $t$ and the derivative of a function by the function with the variable as a subscript with respect to which the derivative is taken.

*Environmental quality*

Environmental quality $E_{t+1}$ represents the state of nature and is reduced by emissions that come from consumption, $c_t$, while it is increased through abatement, $a_t$. The law of motion is given by

$$E_{t+1} = g(E_t, c_t) + \gamma a_t, \tag{1}$$

where $g_E \geq 0$, and $g_c < 0$. We interpret $g_t$ as the state of nature that the generation born at time $t$ inherits. The coefficient $\gamma > 0$ on abatement defines how effective a unit of abatement is in improving environmental quality.

*Environmental culture*

We assume that environmental culture at time $t$ is represented by $X_t$, increased by investments $x_t$ through a production function $\psi(X_t, x_t)$ and depreciates partly during the course of each generation. Function $\psi(X_t, x_t) > 0$ then transforms one unit of expenditure $x_t$ into $\psi(X_t, x_t)$ units of environmental culture. Thus, the intergenerational transmission channel is given by the effect of $X_t$ on $\psi(X_t, x_t)$, while the costly education channel is given by the effect of $x_t$. We define the domain of $X_t$ to be $X_t \geq \psi_0$. The production function is assumed to have the following shape.

**Assumption 1** *Function* $\psi(X_t, x_t) > 0$, $\psi_x(X_t, x_t) > 0$, $\psi_{xx}(X_t, x_t) \leq 0$, $\psi_X(X_t, x_t) > 0$, $\psi_{xX}(X_t, x_t) \leq 0$, $\psi(X_t, 0) < X_t$ *for* $X_t > \psi_0$.

As a result, environmental culture develops according to

$$X_{t+1} = \psi(X_t, x_t). \tag{2}$$

This definition of environmental culture is sufficiently general to allow both a costless intergenerational transmission of environmental culture (albeit an incomplete one), and at the same time for culture-building through costly measures.

Our assumption shall be that, over time, environmental culture will diminish unless costly effort is undertaken to impact it, which implies $\psi(X_t, 0) < X_t$. Thus, without investments, environmental culture will diminish over time to its minimum level, $\psi(\psi_0, 0) = \psi_0$.[10]

The assumption of a non-positive cross-derivative of the culture-generating function, namely $\psi_{xX} \leq 0$, is not innocent. It means that, for higher levels of intergenerationally-transmitted environmental culture, the marginal impact of additions to culture through costly

[10] For example, assume $X_{t+1} = \delta X_t + (1 - \delta)(\psi_0 + \psi_1 x_t)$, with $\delta \in (0, 1)$, then assuming $x_t = 0$, $\forall t$, gives a steady state of $X_\infty = \psi_0$.

measures like education either becomes smaller and smaller or is constant. Thus, this condition states that intergenerationally transmitted culture and educational measures are (weak) substitutes in the development of environmental culture.[11]

*The generations*

We assume that in each period there exist two generations, a young one and an old one. Each generation is represented by a single agent that lives for two periods, called young in the first period of life and old in the second period. We simplify by assuming away population growth. Young agents receive a labor income $w_t \geq 0$. They may spend this income either on capital formation $s_t \geq 0$ for consumption $c_{t+1}$ when old, on abatement $a_t \geq 0$ for the enjoyment of environmental quality when old, or they may invest $x_t \geq 0$ in environmental culture that shapes the trade-off between consumption and environmental quality.[12] As old agents are not altruistic with respect to future generations they use their returns on savings for consumption only.

The utility function of the representative agent takes the form

$$u(X_{t+1}, c_{t+1}, E_{t+1}) = \min\{X_{t+1}c_{t+1}, E_{t+1}\}. \tag{3}$$

This specification, as explained in the previous section, rests on two assumptions. One, society allocates resources based on a Leontieff utility function. Two, the degree to which a unit of consumption is turned into effective consumption depends on a generation's environmental culture. In other words, the predominant environmental culture at the time of decision-taking affects the relative valuation of consumption and environmental quality. This point is closely related to the arguments in Lockwood (1996), Gowdy (1997) or Spash (2000).

In order to provide some intuition to this multiplicative factor, assume $\psi_0 = 0.2$. In this case, five units of consumption are effectively viewed equivalent to one unit of environmental quality. This, for example, could be interpreted as a consumption-oriented society. Assume now that a generation invests $x > 0$ into environmental culture, such that $\psi(\psi_0, x) = 1$. Then one unit of consumption produces the same amount of utility as one unit of environmental quality. In other words, environmental quality becomes relatively more important for utility than before.

The constraints faced by the agent are as follows. He receives wages $w_t > 0$ which he can allocate to savings $s_t \geq 0$, to abatement $a_t \geq 0$, or to investing in environmental culture $x_t \geq 0$. Savings today receive an interest rate of $R_{t+1} > 1$, and the agent fully consumes his

[11]The other case of complements would imply $\psi_{xX} > 0$, suggesting that higher levels of intergenerationally transmitted culture support the development of an environmental culture through e.g. educational measures. We will discuss the implication of this when it is relevant.

[12]The assumption of obtaining utility when old is taken for both simplicity and comparison to the established literature (c.f. John and Pecchenino 1994). Both the dynamics and possible steady states will be affected if one e.g. allows the young generations' consumption to enter the utility function, too. Hence, the way one generalizes the utility function may then affect the results.

savings plus the interest obtained when old. In addition, he faces the law of motion for the environment, as given by equation (1). The constraints can then be summarized as follows.

$$\begin{aligned} w_t &= s_t + x_t + a_t, & (4) \\ c_{t+1} &= R_{t+1} s_t, & (5) \\ E_{t+1} &= g(E_t, c_t) + \gamma a_t. & (6) \end{aligned}$$

Having defined the basic structure of the model, we now take a look as to what are the implications for the basic relationship between environmental culture, the state of economic development and environmental quality.

### 3.1 The temporal equilibrium

We call the equilibrium resulting from the assumptions and conditions above the temporal equilibrium. It is defined as follows.

**Definition 1** *The* temporal equilibrium *consists of the allocations* $\{s_t, a_t, x_t\}$*, where at every* $t = 0, 1, 2, ...,$ *the young generation maximizes* (3) *subject to* (2), (4), (5) *and* (6), *with* $w_t$, $E_t$, $X_t$, $R_{t+1}$ *and* $c_t$ *given.*

Our strategy shall be to study the temporal equilibrium first and then to provide additional assumptions that allow us to investigate the full dynamics of the model. As a preliminary step, we build the intuition of the model.

Thinking about the mathematical problem ex ante, then for interior choices the optimal solution would be to find a combination between $x_t$, $a_t$ and $s_t$ such that

$$E_{t+1} = \psi(X_t, x_t) c_{t+1}. \tag{7}$$

Re-writing this equation by substituting the constraints (4), (5) as well as the environmental law of motion (6) leads to

$$a_t = \frac{\psi(X_t, x_t) R_{t+1}(w_t - x_t) - g(E_t, c_t)}{\psi(X_t, x_t) R_{t+1} + \gamma}. \tag{8}$$

Here we have an equation of the two endogenous variables $x_t$ and $a_t$ that also implicitly determines $s_t$ (via eq. (4)). We define

$$\Gamma(x_t) \equiv R_{t+1} \psi(X_t, x_t)(w_t - x_t). \tag{9}$$

Function $\Gamma(x_t)$ then represents the *potential* level of culturally-weighted consumption[13]. As one may already see from equation (8), abatement is positive as long as the potential, culture-

[13] Potential here means excluding abatement spending.

weighted consumption level exceeds the given amount of environmental quality at the time at which the agent takes his choices.

The following Lemma describes how the potential culture-weighted consumption depends on investments in environmental culture.

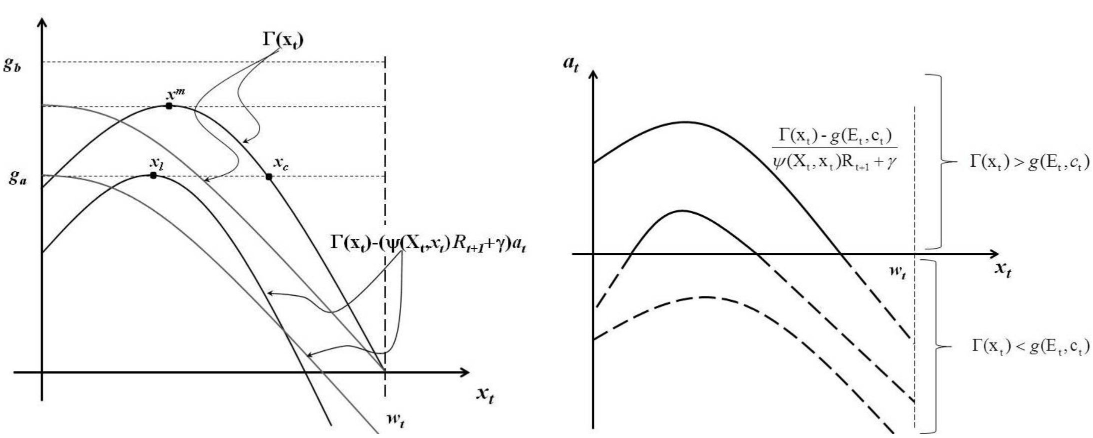
**Lemma 1** *$\exists x_t^m \geq 0$, such that $x_t = x_t^m$ solves*

$$\max_{x_t \in [0, w_t]} \psi(X_t, x_t) R_{t+1}(w_t - x_t).$$

**Proof of Lemma 1** *The derivative of function $\psi(X_t, x_t)R_{t+1}(w_t - x_t)$ with respect to $x_t$ is $\psi_x(X_t, x_t)R_{t+1}(w_t - x_t) - \psi(X_t, x_t)R_{t+1} \equiv \Theta(x_t)$. This function has the properties that $\Theta(0) = \psi_x(X_t, 0)R_{t+1}w_t - \psi(X_t, 0)R_{t+1}$, while $\Theta(w_t) = -\psi(X_t, w_t)R_{t+1} < 0$, and $\Theta'(x_t) = \psi_{xx}(X_t, x_t)R_{t+1}(w_t - x_t) - 2\psi'(x_t)R_{t+1} < 0$. Consequently, an interior solution requires $w_t > \psi(X_t, 0)/\psi_x(X_t, 0)$, in which case $x_t = x_t^m$, where $x_t^m$ is the solution to $\Theta(x_t^m) = 0$. If $w_t \leq \psi(X_t, 0)/\psi_x(X_t, 0)$, then there exists no interior solution to $\Theta(x_t^m) = 0$ and the optimal solution is $x_t^m = 0$.* ■

For a given environmental quality at time $t$, $g(E_t, c_t)$, the agent will choose the optimal mix of $a_t$, $s_t$ and $x_t$. Figure 1 illustrates how we can determine the optimal allocations

**Figure 1:** Different optimal allocations




(a) Function $\Gamma(x_t)$ with corner and interior $a_t$

(b) Potential allocations of $a_t$ as a response to choices of $x_t$

starting from equation (7), (8) and (9). Intuitively speaking, an agent faced with a high level of environmental quality like $g_b$ will be unable to increase his utility via investing in environmental quality since the level of the environment is already higher than his potential culture-weighted consumption. Therefore, he will only maximize $\Gamma(x_t) = \psi(X_t, x_t)R_{t+1}(w_t - x_t)$. By Lemma 1 this maximum is given when $x_t = x_t^m$. As the dark black line associated with $\Gamma(x_t)$ in panel (a) shows, for any $g_t$ larger than or equal to $\Gamma(x_t^m)$, this point will be given by $x_t = x_t^m$. Clearly, $x_t^m$ represents the optimal choice for $x_t$ when $a_t = 0$. Obviously, if $\Gamma_x(x_t) \leq 0, \forall x_t \in [0, w_t]$, then $x_t^m = 0$, as depicted by the light grey lines in Figure 1. Hence, a positive amount $x_t$ will only decrease overall utility since the costs of increasing environmental culture outweigh the benefits. Or, in other words, the increase in environmental culture costs the agent too much consumption, so that overall the culture-weighted consumption decreases. For $\Gamma_x(0) > 0$, then the optimal $x_t > 0$, as the black lines in Figure 1 show. In this case, investments in environmental culture increase culture-weighted consumption.

In contrast, assume now that environmental quality at time $t$ is low, at e.g. $g_a$. In this case keeping $a_t = 0$ would potentially waste income. If $\Gamma(x_t^m) > g(E_t, c_t)$, then a positive amount of abatement would increase utility. As the graph shows, the agent will have too much culture-weighted consumption (relative to environmental quality) for all $x_t < x_c$, while too little for all $x_t > x_c$. However, $x_c$ is not an optimal allocation either as the agent can do better. He could invest less in environmental culture which would increase $X_{t+1}c_{t+1}$. The income saved can then be spent on abatement, which increases $E_{t+1}$ by $\gamma a_t$, which is the direct effect of higher abatement on environmental quality. The additional cost of a higher abatement level is the consumption foregone, which is given by $\psi(X_t, x_t)R_{t+1}$. Thus, a positive abatement shifts function $\Gamma(x_t)$ down by an amount $(\psi(X_t, x_t)R_{t+1} + \gamma)a_t$. In line with the discussion, panel (b) of Figure 1 shows that abatement will only be positive if the potential culture-weighted consumption exceeds the given level of environmental quality $g_t$. The trade-off between abatement and investments in environmental culture is then depicted by the black, full lines.

### 3.1.1 Solving the model

We now solve the model analytically at the temporal equilibrium. The agent's objective is to find the combination of $s_t$, $x_t$ and $a_t$ that leads to a maximum of utility given the constraints and initial conditions. By relying on equation (7) and substituting the constraints, we can write

$$E_{t+1} = \frac{R_{t+1}\psi(X_t, x_t)}{\gamma + \psi(X_t, x_t)R_{t+1}} \Big( \gamma(w_t - x_t) + g(E_t, c_t) \Big). \tag{10}$$

We use this as a new, reduced-form utility function,[14] which we denote by

$$W(x_t) = \frac{\psi(X_t, x_t)}{\gamma + \psi(X_t, x_t)R_{t+1}} \Big( \gamma(w_t - x_t) + g(E_t, c_t) \Big). \tag{11}$$

[14] The term $R_{t+1}$ is equivalent to a monotone transformation, so we can neglect it.

The first-order condition[15] of $W(x_t)$ with respect to $x_t$ gives

$$\psi_x(X_t, x_t)\,(\gamma(w_t - x_t) + g(E_t, c_t)) \leq \psi(X_t, x_t)\big(\gamma + \psi(X_t, x_t)R_{t+1}\big), \tag{12}$$

which holds with equality if $x_t > 0$. This condition states that, for interior solutions, a marginal increase in $x_t$ will increase utility through its effect on the cultural weight, while the marginal cost is a reduction in consumption (because of lower savings), and a decrease in environmental quality (because of lower abatement).

The second-order derivative is

$$\begin{aligned} W_{xx}(x_t) \;\; = \;\; & \big(\psi_{xx}(X_t, x_t)(\gamma(w_t - x_t) + g_t) - 2\gamma\psi_x(X_t, x_t)\big)(\gamma + \psi(X_t, x_t)R_{t+1}) \\ & -2\gamma\psi_x(X_t, x_t)^2 R_{t+1}(\gamma(w_t - x_t) + g_t) < 0, \end{aligned}$$

thus a maximum is assured.

We now define two bounds for environmental quality that are important for the latter analysis and which identify regions in which different optimal regimes will apply.

**Definition 2** *We define the thresholds*

$$g_L \equiv \frac{\psi(X_t, 0)}{\psi_x(X_t, 0)}(\gamma + \psi(X_t, 0)R_{t+1}) - \gamma w_t,$$

*and*

$$g_H \equiv \frac{\psi(X_t, w_t)}{\psi_x(X_t, w_t)}(\gamma + \psi(X_t, w_t)R_{t+1}).$$

As we shall show later, given Assumption 1 it is always true that $g_L < g_H$. Thus, $g_L$ denotes a lower bound, while $g_H$ defines an upper bound.

In Proposition 1 we introduce the optimal choices of the representative agent given the maximization problem that we defined above.

**Proposition 1** *At the temporal equilibrium, the model as described in equations* (2) *to* (6) *leads to at maximum four different regimes that arise depending on the following parameter conditions:*

Regime 1. *If* $g(E_t, c_t) \leq g_L \wedge g(E_t, c_t) < \psi(X_t, 0)R_{t+1}w_t$, *then there exists a unique* $a^*_{1t} = \frac{\psi(X_t,0)R_{t+1}w_t - g(E_t,c_t)}{\gamma+\psi(X_t,0)R_{t+1}}$, $s^*_{1t} = w_t - a^*_t$, *and* $x^*_{1t} = 0$.

Regime 2. *If* $g(E_t, c_t) \geq \psi(X_t, 0)R_{t+1}w_t \wedge w_t \leq \psi(X_t, 0)/\psi_x(X_t, 0)$ , *then* $a^*_{2t} = x^*_{2t} = 0$ *and* $s^*_{2t} = w_t$.

Regime 3. *If* $g(E_t, c_t) \in (g_L, \Gamma(x^m_t)) \wedge w_t > \psi(X_t, 0)/\psi_x(X_t, 0)$, *then there exists a unique* $x^*_{3t} \in (0, w_t)$ *that solves equation* (12), *with* $a^*_{3t} = \frac{\Gamma(x^*_{3t}) - g(E_t,c_t)}{\gamma+\psi(X_t,x^*_{3t})R_{t+1}}$ *and* $s^*_{3t} = w_t - a^*_{3t} - x^*_{3t}$.

[15] One can, equivalently, arrive at this result via maximizing $\psi(X_t, x_t)c_{t+1}$ subject to $\psi(X_t, x_t)c_{t+1} = E_{t+1}$. When substituting the constraints one can then solve for $a_t$ and substitute this into $\psi(X_t, x_t)c_{t+1}$. This will lead to the same first-order condition as equation (12).

Regime 4. *If* $\Gamma(x_t^m) \leq g(E_t, c_t) \wedge w_t > \psi(X_t, 0)/\psi_x(X_t, 0)$*, then* $a_{4t} = 0$ *and* $s_{4t} = w_t - x_{4t}$*, where* $x_{4t}$ *is given by* $x_{4t} = x_t^m$.

**Proof of Proposition 1** *See Proof of Proposition 1 in the Appendix.*

The model here gives rise to a maximum of four different regimes. Which regime turns out to be the optimal one for society then depends on the given economic and environmental conditions at the time of choice. For low wealth levels, society is unable to free resources for environmental culture. Furthermore, if environmental quality is sufficiently high, then society will find it optimal to simply direct all income towards savings and thus maximizes consumption (Regime 2). This is a result of the Leontieff preferences.[16] Based on the maxi-min choice criterion, a relatively high level of environmental quality would make society waste income if it is directed towards increasing environmental quality. Consequently, income is spent on increasing the most needed component of utility, namely culture-weighted consumption, a result akin to Maslow's theory of needs. This Regime 2 emphasizes the zero maintenance, minimum culture (temporal) equilibrium as a result of a low wealth and high environmental quality combination.

In contrast, if environmental quality is relatively low and wealth is not sufficiently high (Regime 1), then society should also target its wealth towards investments in environmental quality. The choices undertaken by society in this regime are then very close to those that are obtained via a utility function that does not place so much emphasis on basic needs (c.f. John and Pecchenino 1994).

When environmental quality and wealth are both sufficiently high, then society may find it optimal to temporarily over-invest (vis-à-vis its steady state level) in environmental culture (Regime 4). This regime is characterized by an abundant environmental quality and wealth. This is optimal until environmental quality is decreased to a level from which onwards it is worthwhile for society to also invest in environmental quality (Regime 3). In other words, if there is no urgent need for society to improve environmental quality, then society will either invest in an environmental culture if it can afford to do so, or not invest in case it is too poor. This result is related to the view of cultural ecology, as e.g. defined in Berry (1975). In this line of literature, it is emphasized that the environment at a given point in time determines both culture and behavior. While we show this to be true at a given point in time, we also find that environmental culture is able to affect future environmental quality. Thus, there exist feedback loops between both the environment and culture.

We now look at some comparative statics using equation 12 assuming we are in Regime 3, such that we have an interior solution to $x_t$ and $a_t$. In this case, changes in the efficiency

[16] Nevertheless, we can show that this result continues to hold even in case of more general preferences, as we discuss below.

of maintenance lead to

$$\frac{dx^*_{3t}}{d\gamma} = \frac{\psi(X_t, x^*_{3t}) - \psi_x(X_t, x^*_{3t})(w_t - x^*_{3t})}{\Omega} > 0,$$

where $\Omega = \psi_{xx}(X_t, x^*_{3t})(\gamma(w_t - x^*_{3t}) + g_t) - 2\psi_x(X_t, x^*_{3t})(\gamma + \psi(X_t, x^*_{3t})R_{t+1}) < 0$. Based upon Lemma 1, it is easy to see that $x^*_{3t} < x^m_t$, and therefore $\psi(X_t, x^*_{3t}) < \psi_x(X_t, x^*_{3t})(w_t - x^*_{3t})$. The denominator is the second-order condition (13) of the maximization problem (11) and is negative. Consequently, increases in the efficiency of maintenance lead to increases in the optimal expenditure on environmental culture.

In case the given level of environmental quality is larger, then investments in environmental culture react according to

$$\frac{dx^*_{3t}}{dg_t} = -\frac{\psi_x(X_t, x^*_{3t})}{\Omega} > 0.$$

Thus, for higher levels of environmental quality, and for lower consumption levels of the previous generation,[17] the current generation will profit from increasing investments in environmental culture. This arises because a higher given level of environmental quality allows to increase the agent's indirect utility. Consequently, the agent will raise culture-weighted consumption, and one means to do so is via increases in environmental culture.

The intergenerationally transmitted level of environmental culture also plays a role for the optimal choice of the young generation's expenditure on environmental culture, and it is given by

$$\frac{dx^*_{3t}}{dX_t} = \frac{\psi_X(X_t, x^*_{3t})(\gamma + 2\psi(X_t, x^*_{3t})R_{t+1}) - \psi_{xX}(X_t, x^*_{3t})(\gamma(w_t - x_t) + g_t)}{\Omega} < 0.$$

This result rests on two parts. One, a higher level of intergenerationally transmitted culture implies that the young generation does not need to invest so much into culture in order to equalize culturally-weighted consumption and environmental quality. Two, since higher levels of transmitted culture reduce the marginal impact of costly educational expenditures, then this provides an additional incentive for further reductions in investments in culture.[18]

In addition, a wealthier agent will change his investments in environmental culture according to

$$\frac{dx^*_{3t}}{dw_t} = -\frac{\gamma\psi_x(X_t, x^*_{3t})}{\Omega} > 0.$$

Thus, he will increase environmental culture, and one reason is simply that higher wealth frees more of the agent's income for investments in environmental culture.

As a remark, in the extreme case of $\psi(X_t, 0) = 0$, the lower bound $g_L$ will be given by

[17] Taking the derivative with respect to $g_t$ is a short-hand notation for $dg_t = g_E dE_t + g_c dc_t$.

[18] This is clearly the result of assuming $\psi_{xX} \leq 0$. If one were to assume instead sufficiently strong complementarity, then this may even imply a positive relation between $X_t$ and $x_t$. Consequently, this result emphasizes the need to further empirically investigate the relationship between intergenerationally transmitted environmental culture and culture adopted through a costly learning process like environmental education.

$g_L = 0$. In this case, both Regimes 1 and 2 will not exist, and there will always be an interior solution in $x_t$. Consequently, only Regimes 3 and 4 apply. The existence of a corner solution in $x_t = 0$, and therefore whether or not society may optimally find itself in Regime 1 or 2, thus depends on whether or not there is a minimal, positive amount of environmental culture at each point in time.

We also note that the bounds identifying the four different regimes are time-varying. They are an implicit function of the interest rate, the wages, the intergenerationally-transmitted environmental culture, and, in the case of the bound $\Gamma(x_t^*)$ which we obtained from the first-order condition equation (12), also depend on the given environmental quality and consumption. Consequently, the regions that make up the four regimes depend on the level of economic development, the existing stock of culture, and the condition of the environment. How these bounds evolve over time, therefore, depends on the intertemporal structure of the model, which we define in section 3.2.

### 3.2 The intertemporal equilibrium

In the previous section we derived the optimal decisions of a representative agent for a given return on capital and for given wages. Consequently, we could only analyze the effect of the agent's choices for a given point in time. We now extend the results in the previous section by assuming a law of motion for capital, $k_t \geq 0$. In doing so we can derive the evolution of our economy over time and study its potential convergence to a steady state. In addition to introducing how capital accumulates, we also provide a more explicit structure to the model, which vastly facilitates the subsequent analysis at a minimal loss of generality.

**Assumption 2** *Capital accumulation is given by* $s_t = k_{t+1}$.

Capital is assumed to depreciate fully within the course of one generation. This assumption is borrowed from de la Croix and Michel (2002). It approximately corresponds to estimates of the speed of capital depreciation during one generation.

*The representative firm*

We assume the existence of a representative firm that produces under a neoclassical, constant returns to scale technology, with capital $K$ and labor $L$ as inputs, given by $F(K, L) = F(k, 1)L = f(k_t)L$, where $k = K/L$. We normalize labor to 1. It hires labor at wage $w_t$ and rents capital at rate $R_{t+1}$ from the young generation, that then is used in production of a product that is sold on a competitive market with perfect foresight. As a result, interest payments are $R_{t+1} = R(k_{t+1}) = f'(k_{t+1})$, and wage payments will be $w_t = w(k_t) = f(k_t) - f'(k_t)k_t$. Since consumption is given by $c_{t+1} = R_{t+1}s_t$, then this also implies $c_{t+1} = f'(k_{t+1})k_{t+1}$.

**Assumption 3** *For all* $k_t > 0$*, the production function takes the form* $f(k_t) = k_t^\alpha > 0$, $\alpha \in (0, 1)$.

We normalize Total Factor Productivity to one for simplicity. Based on this explicit production function, the wage constraint and the law of motion for capital accumulation, then the maximum feasible, constant level of capital is given by $k^{\max} = (1-\alpha)^{\frac{1}{1-\alpha}}$.[19]

**Assumption 4** *We define $\eta \equiv (1-\alpha)\gamma - \alpha\beta$ and assume that $\eta > 0$.*

This is a simple feasibility condition, which means that the improvement to environmental quality from the maximum possible abatement expenditure must be able to exceed the consumption externality. Without this condition, no steady state in environmental quality will ever be achieved.

In addition, we take a linear structure for function $g_t$, with $g(E_t, c_t) = E_t - \beta c_t$, where $\beta > 0$ denotes emissions per unit consumption. Consequently, the evolution of environmental quality is as follows.

**Assumption 5** *For all $E_{t+1} > 0$, the environment evolves according to $E_{t+1} = E_t - \beta c_t + \gamma a_t$. For $E_t - \beta c_t + \gamma a_t \leq 0$, we assume the lower bound $E_{t+1} = 0$, $\forall \tau \geq t$.*

On the relevant time-scale of mankind, the own regeneration rate of the environment is assumed to be negligible. Thus, in contrast to John and Pecchenino (1994), environmental quality does not return to its natural level in case there is no human interference. One could allow for a natural regeneration rate, as in e.g. Jouvet et al. (2005). However, our focus is on studying whether investments should be directed towards abatement or altruism, and therefore we want to simplify the theoretical framework as much as possible and necessary. We discuss the implication of this assumption in footnote 21.

Based on these specifications we can now define and derive the intertemporal equilibrium.

**Definition 3** *Given the capital stock $k_0 > 0$, the stock of environmental culture $X_0 \geq \psi_0$, and the environmental quality $E_0 > 0$, an* intertemporal equilibrium *is a temporal equilibrium that furthermore satisfies, for all $t \geq 0$, the capital accumulation condition $k_{t+1} = s_t$, interest payments $R_{t+1} = R(k_{t+1}) = f'(k_{t+1})$, and wage payments $w_t = w(k_t) = f(k_t) - f'(k_t)k_t$, as well as Assumptions 1 to 5.*

The objective now is to investigate the intertemporal equilibrium for each of the four regimes that we identified above.

### 3.2.1 Regime 1

From the conditions at the temporal equilibrium we know that Regime 1 applies if $g(E_t, c_t) \leq g_L \wedge g(E_t, c_t) < \psi(X_t, 0)R_{t+1}w_t$. As a result there exists a unique $a^*_{1t} = \frac{\psi(X_t,0)R_{t+1}w_t - g(E_t,c_t)}{\gamma + \psi(X_t,0)R_{t+1}}$, $s^*_{1t} = w_t - a^*_t$, and $x^*_{1t} = 0$.

[19] In the subsequent analysis we shall concentrate on the case where $k_t \in [0, k^{\max}]$, although the results are also applicable outside of this domain for non-negative $k_t$.

This gives rise to the dynamic system

$$\begin{aligned} k_{t+1} &= \frac{E_t + \eta k_t^{\alpha}}{\gamma + \psi(X_t, 0)\alpha k_{t+1}^{\alpha-1}}, & (13) \\ E_{t+1} &= \psi(X_t, 0)\alpha k_{t+1}^{\alpha}, & (14) \\ X_{t+1} &= \psi(X_t, 0). & (15) \end{aligned}$$

As one can already see, environmental culture converges monotonically to its steady state, while environmental quality depends solely on the evolution of capital (and trivially on culture). Hence, the dynamics of Regime 1 will hinge on the convergence of capital. The dynamic system gives rise to a steady state characterized by the equations

$$\begin{aligned} E &= \psi_0 \alpha k^{\alpha} \equiv z_1(k), & (16) \\ E &= \gamma k + (\psi_0 \alpha - \eta) k^{\alpha} \equiv z_2(k), & (17) \\ X &= \psi_0. & (18) \end{aligned}$$

The steady state and the dynamics in Regime 1 can be summarized as follows.

**Proposition 2** *In Regime 1 there exists a steady state if* $\psi_x \eta^{\frac{1}{1-\alpha}} < \psi_0 \gamma^{\frac{1}{1-\alpha}}$. *This unique steady state is given by* $\{\bar{k}_1, \bar{E}_1, \bar{X}_1\} = \{(\eta/\gamma)^{\frac{1}{1-\alpha}}, \alpha\psi_0 (\eta/\gamma)^{\frac{\alpha}{1-\alpha}}, \psi_0\}$ *and it is locally asymptotically stable. Convergence to the steady state is monotonic.*

**Proof of Proposition 2** *See Proof of Proposition 2 in the Appendix.*

At steady state $\{\bar{k}_1, \bar{E}_1, \bar{X}_1\}$, condition $g_t \leq g_L$ implies $\bar{k}_1 \leq \psi_0/\psi_x$, which is equivalent to the parameter condition $(\eta/\gamma)^{\frac{1}{1-\alpha}} \leq \psi_0/\psi_x$, while condition $g_t < \psi(X_t, 0)R_{t+1}w_t$ always holds.

Regime 1 occurs if the given environmental quality at the time that the agent takes his decision is so low that investments in environmental culture would not be able to increase utility. However, a positive level of abatement would be worthwhile since an increase in environmental maintenance reduces the gap between environmental quality and culture-weighted consumption. This then implies to a convergence to a steady state without investments in environmental culture, but a positive amount of environmental maintenance. The economy will be caught in a low-culture trap, where abatement is undertaken to safeguard a minimum level of environmental quality.

### 3.2.2 Regime 2

If $g(E_t, c_t) \geq \psi(X_t, 0)R_{t+1}w_t \wedge w_t \leq \psi(X_t, 0)/\psi_x(X_t, 0)$, then $a_{2t}^* = x_{2t}^* = 0$ and $s_{2t}^* = w_t$. We can easily show that $g_L \geq \psi(X_t, 0)R_{t+1}w_t$ requires $w_t \leq \psi(X_t, 0)/\psi_x(X_t, 0)$.

In this case the dynamic system reduces to

$$k_{t+1} = (1-\alpha)k_t^{\alpha}, \tag{19}$$

$$E_{t+1} = E_t - \alpha\beta k_t^{\alpha}, \tag{20}$$

$$X_{t+1} = \psi(X_t, 0). \tag{21}$$

Clearly, no steady state $\{\bar{E}_2, \bar{k}_2, \bar{X}_2\}$ exists in this case. The explicit solution to the difference system (19) and (20) can be written as

$$k_t = (1-\alpha)^{\frac{1}{\alpha}\left(\sum_{\tau=0}^{t}\frac{(1)_\tau a^\tau}{\tau!}-1\right)} k_0^{\alpha^t} \equiv \theta(k_0, t), \tag{22}$$

$$E_t = E_0 - \alpha\beta\sum_{\tau=0}^{t}\theta(k_0,\tau), \tag{23}$$

with $(1)_\tau$ being the Pochhammer symbol which denotes the falling factorial $(x)_y = x(x+1)...(x+y-1)$ (see Abramowitz and Stegun 1972). Environmental culture converges to its minimum level independent of the other variables and does not affect system (19) and (20).

This Regime 2 emphasizes zero maintenance and no investment in environmental culture as a result of two sets of conditions that are working together. The first set of conditions states that low wages with a too small investment sensitivity of environmental cultural leads to a corner solution in cultural investments. The second set of conditions states that investments in environmental maintenance do not pay off since the given level of environmental quality already exceeds culture-weighted consumption. With the laws of motion for capital accumulation and environmental quality in mind, we then see that environmental quality is only reduced (due to the zero maintenance), while maximum effort is directed towards capital accumulation. As a result, environmental quality diminishes at an increasing rate due to the growing levels of consumption. Consequently, there cannot exist an intertemporal equilibrium with a steady state in Regime 2. Instead, Regime 2 converges into either Regime 1 or Regime 4. It converges into Regime 4 if capital accumulates faster than environmental quality is destroyed, which would, for example, be the case for a sufficiently low emission rate (low $\beta$) or a high share of capital in production (high $\alpha$).

### 3.2.3 Regime 3

If $g(E_t, c_t) \in (g_L, \Gamma(x_t^m)) \wedge w_t \geq \psi(X_t, 0)/\psi_x(X_t, 0)$, then there exists a unique $x_{3t}^* \in (0, w_t)$ that solves equation (12), with $a_{3t}^* = \frac{\Gamma(x_{3t}^*) - g(E_t, c_t)}{\gamma + \psi(X_t, x_{3t}^*)R_{t+1}}$ and $s_{3t}^* = w_t - a_{3t}^* - x_{3t}^*$. As a result, Regime 3 is the only regime with interior solutions in both abatement and investments in environmental culture.

The equations that are describing the dynamic system in this case are given by

$$\begin{aligned}
\eta k_t^\alpha + E_t - \gamma x_t &= \frac{\psi(X_t, x_t)}{\psi_x(X_t, x_t)}\big(\gamma + \psi(X_t, x_t)\alpha k_{t+1}^{\alpha-1}\big), &(24)\\
E_{t+1} &= \psi(X_t, x_t)\alpha k_{t+1}^\alpha, &(25)\\
k_{t+1} &= (1-\alpha)k_t^\alpha - x_t - a_t, &(26)\\
E_{t+1} &= E_t - \alpha\beta k_t^\alpha + \gamma a_t, &(27)\\
X_{t+1} &= \psi(X_t, x_t). &(28)
\end{aligned}$$

Equation (24) comes from the first-order condition (12), equation (25) is equation (7) that derives directly from the maximin criterion, while equation (26) is the capital accumulation equation, (27) the law of motion for environmental quality and (28) the evolution of environmental culture.

We define $X = \tilde{\psi}(x_3^*)$ as the steady state level of environmental culture for a given optimal solution of $x_3^*$,[20] and we define $\zeta(k) \equiv \eta/\gamma k^\alpha - k (= x_3^*)$. Then the steady state of this dynamic system is characterized by

$$\begin{aligned}
k &= \frac{\psi\big(\tilde{\psi}(\zeta(k), \zeta(k)\big)}{\psi_x\big(\tilde{\psi}(\zeta(k)), \zeta(k)\big)}, &(29)\\
E &= \psi\big(\tilde{\psi}(\zeta(k)), \zeta(k)\big)\alpha k^\alpha, &(30)\\
X &= \psi\big(\tilde{\psi}(\zeta(k)), \zeta(k)\big). &(31)
\end{aligned}$$

**Proposition 3** *There exists at least one steady state in Regime 3 if equations* (29), (30) *and* (31) *hold and if condition $\psi_x(\psi_0, 0)\eta^{\frac{1}{1-\alpha}} > \psi_0\gamma^{\frac{1}{1-\alpha}}$ is satisfied.*

**Proof of Proposition 3** *See Proof of Proposition 3 in the Appendix.*

The implicit nature of function $\psi(X, x)$ makes it difficult to provide conditions on the uniqueness of equilibria and stability. As can be easily seen from the steady state equations, if this function is strongly non-linear, then several steady states may exist. However, sufficient linearity in the $\psi(X, x)$ function allows to study the equilibria further. We do this in section 3.3.

### 3.2.4 Regime 4

If $\Gamma(x_t^m) \leq g(E_t, c_t) \wedge w_t > \psi(X_t, 0)/\psi_x(X_t, 0)$, then $a_{4t} = 0$ and $s_{4t} = w_t - x_{4t}$, where $x_{4t}^* = x_t^m$.

Threshold $x_t^m$ implies that $x_t^m = (1-\alpha)k_t^\alpha - \psi(X_t, x_t)/\psi_x(X_t, x_t)$, where $x_t^m$ is implicitly

[20] The existence of this function derives from the continuity and differentiability of $\psi(X, x)$ and the Implicit Function Theorem.

defined by $X_t$ and $k_t$. This leads to the dynamic system

$$E_{t+1} = E_t - \alpha\beta k_t^\alpha, \quad (32)$$
$$k_{t+1} = (1-\alpha)k_t^\alpha - x_t^m, \quad (33)$$
$$X_{t+1} = \psi(X_t, x_t^m). \quad (34)$$

This holds until either $g_t < \Gamma(x_t^m)$ or $(1-\alpha)k_t^\alpha \leq \psi/\psi_x$. We conclude that there exists no steady state in Regime 4.[21]

In Regime 4, environmental quality is so high that it does not pay off to invest in maintenance. At the same time, income is sufficiently high for agents to spend a part of it on investments in environmental culture. As a result, environmental quality is diminished over time. Regime 4 then converges in either of the three other regimes.

### 3.3 A special case

In this section we provide an explicit functional form for the law of motion of environmental culture which allows us to obtain further results, and to explicitly study the existence and convergence to equilibria especially for Regime 3 at the intertemporal equilibrium.

**Assumption 6** *For all $x_t > 0$, the culture function assumes $\psi(X_t, x_t) = \psi_0 + \psi_1 x_t > 0$, with $\psi_0 > 0$ and $\psi_1 > 0$.*

The environmental culture function attains its minimum at $\psi_0 > 0$, and increases linearly with investments in environmental culture at rate $\psi_1 > 0$. This functional form implies full depreciation of environmental culture during the course of one generation. In other words, the channel of intergenerationally transmitted environmental culture is absent. It can easily be shown that, as long as intergenerationally transmitted culture appears as a linear addition in function $\psi$, then this has no repercussions for the steady state level.[22]

Using Assumption 6, we can now draw further results for the existence of and convergence to potential steady states in Regime 3.

[21]This result crucially hinges on the absense of a natural regeneration rate in environmental quality. For example, if we were to assume a law of motion for environmental quality à la Jouvet et al. (2005), which could be given by $E_{t+1} = m\tilde{E} + (1-m)E_t - \beta c_t + \gamma a_t$ where $m \in (0,1)$ is the rate of convergence to the natural state $\tilde{E} > 0$, then while Regime 1 and 3 stay qualitative the same, there may be a qualitative difference in Regime 2 and 4. If $\tilde{E} > \frac{\alpha\beta}{m}k^\alpha$, then there may exist a locally asymptotically stable steady state. Thus, for a low natural level of the environment, or a sufficiently slow regeneration rate, we find that our previous results continue to hold qualitatively even in the case of a more general law of motion for environmental quality.

[22]For example, let us define $\psi(X_t, x_t) = \delta X_t + (\psi_0 + \psi_1 x_t > 0)(1-\delta)$, where $\delta \in (0,1]$ is the relative importance of intergenerationally transmitted culture versus costly education, then the steady state, for a generic $x$, will be given by $X = \psi_0 + \psi_1 x$. This is precisely the same steady state as would arise from the explicit law of motion as defined in Assumption 6.

Solving equation (24) gives the interior solution[23] for $x_t$ by

$$\begin{aligned} x_{3t}^* &= \frac{1}{\alpha\psi_1 k_{t+1}^{\alpha-1}}\Big[-\left(\gamma + k_{t+1}^{\alpha-1}\alpha\psi_0\right) \\ &\quad + \sqrt{\gamma^2 + \alpha k_{t+1}^{\alpha-1}\left(\gamma\psi_0 + E_t\psi_1 + \psi_1\eta k_t^\alpha\right)}\Big], \qquad (35) \\ &\equiv \rho(E_t, k_t, k_{t+1}). \qquad (36) \end{aligned}$$

Condition $g_t > g_L$ assures that $x_3^* > 0$. Then we use equations (26) and (27) to substitute out $a_t$, (25) to substitute out $E_{t+1}$, and substitute the optimal solution $x_{3t}^* = \rho(E_t, k_t, k_{t+1})$. Thus, the system describing the dynamic evolution of case 3 is given by

$$\begin{aligned} E_{t+1} &= (\psi_0 + \psi_1\rho(E_t, k_t, k_{t+1}))\alpha k_{t+1}^\alpha, \qquad (37) \\ \psi_0\alpha k_{t+1}^\alpha + \gamma k_{t+1} &= E_t + \eta k_t^\alpha \\ &\quad -(\psi_1\alpha k_{t+1}^\alpha + \gamma)\rho(E_t, k_t, k_{t+1}). \qquad (38) \end{aligned}$$

As shown above, condition $g_L < g_H$ is always satisfied. Condition $g_t > g_L$ leads to $(\psi_1 k_{t+1} - \psi_0)(\psi_0\alpha k_{t+1}^{\alpha-1} + \gamma) > \psi_1(\psi_1\alpha k_{t+1}^\alpha + \gamma)\rho$. A necessary condition for $g_t > g_L$ to hold is thus $k_{t+1} > \psi_0/\psi_1$. The time-constant versions of equations (37) and (38) are given[24] by

$$\begin{aligned} E &= \gamma\left(2k - \frac{\psi_0}{\psi_1}\right) + (\alpha k\psi_1 - \eta)k^\alpha \equiv w_1(k), \qquad (39) \\ E &= \frac{1}{2}\Big[-2\gamma k + \alpha k^{\alpha+1}\psi_1 \\ &\quad + \sqrt{k(k(\alpha\psi_1 k^\alpha - 2\gamma)^2 + 4\alpha k^\alpha(\gamma\psi_0 + \eta\psi_1 k^\alpha))}\Big] \equiv w_2(k). \qquad (40) \end{aligned}$$

The steady state is then obtained by combining the equations (24) to (27) and is given by

$$\begin{aligned} &\left(\gamma + \left(\psi_0 + \psi_1\left[\frac{\eta}{\gamma}k^\alpha - k\right]\right)\alpha k^{\alpha-1}\right)k \\ &= \frac{\psi_0 + \psi_1\left[\frac{\eta}{\gamma}k^\alpha - k\right]}{\psi_1}\left(\gamma + \left(\psi_0 + \psi_1\left[\frac{\eta}{\gamma}k^\alpha - k\right]\right)\alpha k^{\alpha-1}\right). \qquad (41) \end{aligned}$$

**Proposition 4** *There exists a unique steady state* $\{\bar{k}_3, \bar{E}_3\}$ *to the dynamic system* (37) *and* (38) *given by equation* $2\psi_1\bar{k}_3 = \psi_0 + \psi_1\frac{\eta}{\gamma}\bar{k}_3^\alpha$ *and* $\bar{E}_3 = \alpha\bar{k}_3^{\alpha+1}\psi_1$*, if* $\bar{k}_3 > \psi_0/\psi_1$*. This steady state is locally asymptotically stable and approached monotonically.*

**Proof of Proposition 4** *See Proof of Proposition 4 in the Appendix.*

Regime 3 is the only regime that sees an interior solution in environmental culture that actually converges to a steady state. In this regime, environmental quality $g_t$ is sufficiently

[23]We neglect the negative root.
[24]For their derivation see the Appendix.

high so that investments in environmental culture make sense. At the same time, it is not high enough in order to induce a corner solution in maintenance. Or, in other words, the maximum potential level of culture-weighted consumption exceeds the current level of environmental quality, which gives some room for investments in environmental quality. These positive investments in environmental quality help to maintain a level of environmental quality at which it is worthwhile to also invest in environmental culture.

From the analysis above it is clear that either the steady state in Regime 1 gets picked up, or the one of Regime 3. Which one will be asymptotically approached depends on the parameter configurations. This is shown graphically in Figure 2 for two sets of generic parameter conditions. Under the first set of parameters ($\alpha = 0.3$, $\beta = 1.5$, $\gamma = 2$, $\psi_0 = 0.4$, $\psi_1 = 1$), the steady state in Regime 1 gets selected, while under the second set of parameter conditions ($\alpha = 0.3$, $\beta = 1$, $\gamma = 2.2$, $\psi_0 = 0.4$, $\psi_1 = 1$), it is the steady state in Regime 3 that is approached asymptotically. The analytical derivations for the general shape of the regions are available in the Appendix. We denote the steady state curves of Regime 1 as $E = z_1(k)$ and $E = z_2(k)$, while those of Regime 3 as $E = w_1(k)$ and $E = w_2(k)$. They are depicted as the dashed lines in Figure 2. The boundaries of the four regions are defined according to the conditions in Proposition 1 together with the explicit functional forms and the law of motion for capital accumulation. Whether the dynamics in Regime 4 converge necessarily to Regime 3 or whether they may also converge to Regime 2 depends on whether condition $1-\alpha \geq (\psi_0/\psi_1)^{1-\alpha}$ applies or not.[25] If it applies, then $k_t$ and $E_t$ converge from Regime 4 into Regime 3, otherwise they may also converge to Regime 2. The arrows depict the dynamics for the case of $1 - \alpha \geq (\psi_0/\psi_1)^{1-\alpha}$.

We should note that the general shape of the functions that separate the four regions will be preserved even under an environmental culture function that is linear or Cobb-Douglas in $X_t$.[26] However, due to the third dimension that the autoregressive form of environmental culture introduces, these functions will not be constant as shown in Figures 2 but will be time-varying.

For matters of comparison, we now define a restricted model.

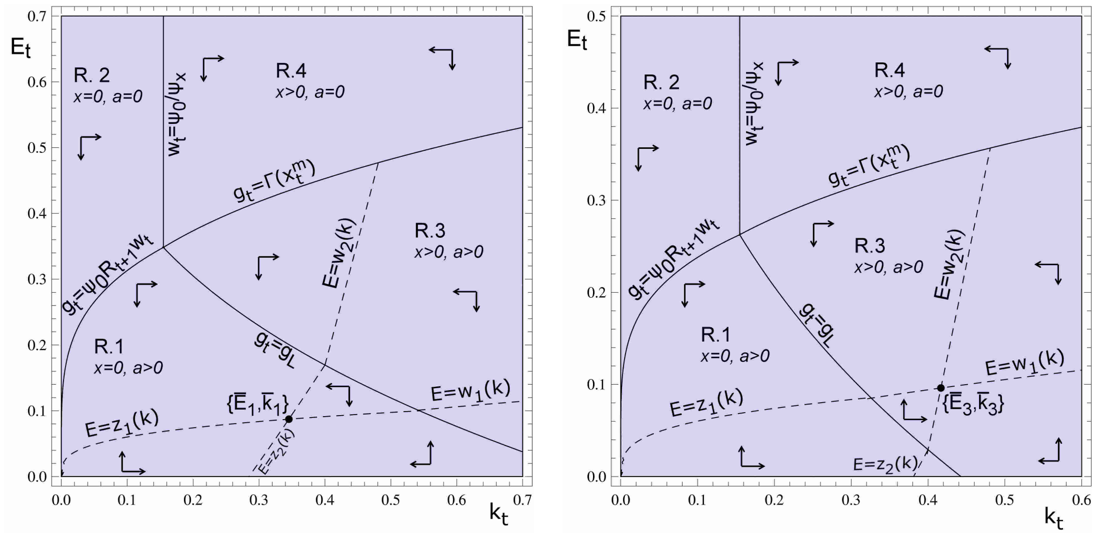
**Definition 4** *The* restricted model *is given by equations* (3) *to* (6) *with* $x_t = 0$, $\forall t$.

Thus, the restricted model is simply the original one introduced above where the young generation cannot invest in environmental culture. As a result, in this restricted model we will always have that $X_t = \psi_0$, $\forall t$.

[25] In Regime 4 we can derive the following. If $1 - \alpha > (\psi_0/\psi_1)^{1-\alpha}$, then $k_{t+1} > (\leq) k_t$ for $k_t < (\geq) \frac{\psi_0+\psi_1(1-\alpha)k_t^\alpha}{2\psi_1}$. Consequently, for $1 - \alpha \geq (\psi_0/\psi_1)^{1-\alpha}$, then the optimal solutions for $k_t$ and $E_t$ will move from Regime 4 to Regime 3 over time. Instead, if $1 - \alpha < (\psi_0/\psi_1)^{1-\alpha}$, then $k_t$ and $E_t$ converge either to Regime 3 or to Regime 2.

[26] An example for a linear function was given in footnote 22. An example for a Cobb-Douglas form with the same steady state properties would be $\psi(X_t, x_t) = X_t^\delta(\psi_0 + \psi_1 x_t)^{1-\delta}$.

**Figure 2:** Different optimal allocations




(a) Steady State in Region 1

(b) Steady State in Region 3

We now clarify the role of the endogenous environmental culture further. Assume that the agent cannot invest in environmental culture. We are, thus, in the case of the *restricted model*. This implies that the optimal decision for abatement is

$$a_t = \frac{\psi_0 R_{t+1} w_t - g(E_t, c_t)}{\gamma + \psi_0 R_{t+1}},$$

if $\psi_0 R_{t+1} w_t > g(E_t, c_t)$, and $a_t = 0$ otherwise. Thus, in case the agent cannot invest in environmental culture, we still recover Regime 1 and 2, but these are not anymore constrained by the other two regimes. Using the explicit functional forms, this choice for $a_t$ then leads to the dynamic system

$$E_{t+1} = \psi_0 \alpha k_{t+1}^{\alpha}, \tag{42}$$

$$k_{t+1} = \frac{E_t + \eta k_t^{\alpha}}{\gamma + \psi_0 \alpha k_{t+1}^{\alpha-1}}. \tag{43}$$

This dynamic system is exactly the same as the one arising in Regime 1, and consequently the steady state and the dynamics are equivalent. As a result, if the agent cannot invest in environmental culture, then he will necessarily pick up the steady state in Regime 1. If the parameter combinations induce the agent who may invest in environmental culture to also choose the steady state in Regime 1, then both cases will lead to the same outcome. However, the difference arises if the agent may freely invest in environment culture and parameter combinations make him pick up the steady state in Regime 3. In this case, the agent will (either initially or eventually) invest in environmental culture, inducing a lower steady state level of capital, but a higher steady state level of environmental quality. This arises since the improvements in environmental culture raise culture-weighted consumption, which induces the agent to reduce savings in favor of investments in environmental culture (and potentially abatement). The reduction in savings leads to a lower steady state capital stock, but also to less consumption and therefore higher steady state environmental quality. We summarize this in the following proposition. We denote the variables of the restricted model in which the young generation cannot invest in environmental culture with a hat.

**Proposition 5** *Assume* $g_t < \psi_0 R_{t+1} w_t$*, then*

- *for* $g_t \leq g_L$*, we have* $\hat{k} = \bar{k}_1 = (\eta/\gamma)^{\frac{1}{1-\alpha}}$*, and* $\hat{E} = \bar{E}_1 = \psi_0 \alpha \bar{k}_1^{\alpha}$*.*
- *for* $g_t \in (g_L, \Gamma(x_t^m))$*, we obtain* $\hat{k} > \bar{k}_3$*, and* $\hat{E} < \bar{E}_3$*.*

**Proof of Proposition 5** *See Proof of Proposition 5 in the Appendix.*

Conclusively, by raising the importance of environmental quality for utility, investments in environmental culture lead to lower levels of consumption and higher environmental quality at steady state in comparison to the restricted model.

Following a similar line of logic we can show that, for a sufficiently high level of environmental quality, environmental culture leads to earlier investments in maintenance expenditure in comparison to the restricted model. Intuitively, since investments in environmental culture raise the importance of environmental quality for utility, then it becomes also more worthwhile for society to invest in maintenance due to the positive feedback between culture and the environment.

The capital-environment combination in which environmental culture leads to positive abatement expenditure in contrast to the restricted model where no abatement expenditure is undertaken is given by $\Gamma(x_{3t}^*) \in (\psi(X_t, 0) R_{t+1} w_t, \Gamma(x_t^m))$. As a result, environmental culture has not only a positive impact on environmental quality through lower levels of consumption, but in addition it improves the environment through maintenance expenditure for capital-environment combinations at which, without environmental culture, no maintenance would be undertaken.

What we, thus, find is that wealth drives utility through higher consumption levels and the possibility to improve environmental quality through maintenance, but after a certain level of economic development there are other aspects that are raised into focus. In our case, the particular aspect is environmental culture. A society that is rich enough will invest in both culture and the environment, and thereby end up with a higher level of utility than one that is unable to invest in environmental culture. This result suggests that, after a certain level of economic development, society may wish to undergo a social change that places cultural aspects, here environmental cultural aspects, at the forefront of decision-taking.

### 3.4 A comment on the utility function

It is obvious that the existence and conditions leading to those four possible regimes should depend on the shape of the utility function. Our crucial assumption here is that environmental culture, on its own, does not raise the utility of an agent. Environmental culture improves an agent's welfare only because it alters his relative perception of consumption and environmental quality while both are required to be in a favorable proportion in the sense that the agent can actually benefit from this change in the relative perception. Thus, even though the Leontieff function that we assume allows for trade-offs, as one can see by the existence of Regime 3, these trade-offs are constrained to wealth-environment combinations that, outside of these combinations, lead to hard constraints.

Some readers may thus wonder whether the results above are preserved for more general utility functions. For example, if one were to believe that environmental culture may alter the relative perception (or use in utility) for any level of consumption and environmental quality, then one may wish to investigate the implication of assuming a utility function that allows for trade-offs even in the limits, as well as for smoother interior trade-offs. At this point is important to note that there are two impacts of environmental culture implicit in any functional form but the Leontieff case. One, there is the trade-off effect, meaning that more environmental culture augments both the marginal benefit of consumption and that of environmental quality. Two, there is the scale effect, meaning that an investment in environmental culture raises the utility of an individual for *any* bundle of culturally-weighted consumption and environmental quality. As a result, a higher level of environmental culture implies that one could achieve the same level of utility with both a lower level of consumption and a lower level of environmental quality. In contrast, in the Leontieff case, investments in environmental culture only have an effect if environmental quality is sufficiently large.

Let us, thus, investigate the potential change that a more general utility function would introduce to our model. Assume the new utility function is given by $V(X_{t+1}c_{t+1}, E_{t+1})$, with $V_c > 0$, $V_E > 0$, $V_{cE} > 0$ and standard concavity and Inada conditions.

Substitution of the constraints (4), (5), (6) as well as (2), then maximizing subject to $x_t$ and $a_t$, leads to two first-order conditions, $\psi_x(w - x - a) \leq \psi$, and $V_c \psi R \geq V_E \gamma$. Assuming that the Implicit Function Theorem can be applied to the second first-order condition, we may solve this for $a_t$. Denote this (interior) solution as $a_t = \rho(w_t, x_t; \mathbf{Z}_t)$, where the vector $\mathbf{Z}_t$ is given by $\mathbf{Z}_t = \{X_t, R_{t+1}, E_t, c_t\}$. Substituting then this solution for $a_t$ into the first-order condition yields

$$\psi_x(X_t, x_t)\big(w_t - x_t - \rho(w_t, x_t; \mathbf{Z}_t)\big) \leq \psi(X_t, x_t). \tag{44}$$

We notice that a more generic utility function only alters the trade-offs for the optimal abatement expenditure, while the optimality condition for investments in culture is essentially unaltered. Hence, we know that environmental culture in the generic utility case will only be different from the Leontieff case because incentives for abatement expenditure may be different. Furthermore, if $\rho(w_t, x_t; \mathbf{Z}_t) = \frac{\psi(X_t,x_t)R_{t+1}(w_t-x_t)-g_t}{\gamma+\psi(X_t,x_t)R_{t+1}}$, then a more generic utility function will lead to the same results as our Leontieff case.

While, in principle, it is possible to do this comparison with the implicit utility function given above, the conditions obtained are rather involved and not too enlightening. Hence, we impose as an explicit functional form the Constant Elasticity of Substitution (CES) case. This functional form incorporates, from perfect complements (Leontieff case) to perfect substitutes (linear case), every conceivable way of trading off culturally-weighted consumption and environmental quality, albeit with the restriction that, as the name suggests, the elasticity of substitution is kept constant. We, thus, impose the functional form $V(X_{t+1}c_{t+1}, E_{t+1}) = \left((X_{t+1}c_{t+1})^\theta + E_{t+1}^\theta\right)^{1/\theta}$, where $\theta \in (-\infty, 1]$, with $\theta = -\infty$ being the Leontieff case, $\theta = 0$ the Cobb-Douglas (or logarithmic) case, and $\theta = 1$ is the linear model.

Utilizing this CES functional form we can then obtain an explicit solution for abatement, which now is given by

$$a_t = \frac{\gamma^{\frac{1}{1-\theta}}\left(\psi(X_t, x_t)R_{t+1}\right)^{\frac{\theta}{\theta-1}}(w_t - x_t) - g_t}{\gamma + \gamma^{\frac{1}{1-\theta}}\left(\psi(X_t, x_t)R_{t+1}\right)^{\frac{\theta}{\theta-1}}} \quad (\geq 0). \tag{45}$$

In the Leontieff case ($\theta = -\infty$) we recover equation (8). The most commonly used utility function in the literature, namely the logarithmic one ($\theta = 0$), leads to an optimal solution for $a_t$ given by $a_t = \frac{1}{2\gamma}(\gamma(w_t - x_t) - g_t)$, which does not directly depend on environmental culture $\psi(X_t, x_t)$ and the interest rate $R_{t+1}$. Finally, the linear case with $\theta = 1$ induces a bang-bang solution in $a_t$. The question that now arises is whether the results derived throughout the article still hold or not. The derivations are available in the online appendix.

It can easily be shown that this may lead to the same four regimes as the Leontieff utility function, albeit with different conditions constraining those regimes. These conditions obviously reflect the slightly different nature of the utility function. Our robustness exercises show that the main results from the Leontieff case are preserved as long as culture-augmented consumption and environmental quality are complements for utility. More specifically, relying upon a constant elasticity of substitution (CES) utility function and assuming that consumption and the environment are complements, we find the following. The conditions that separate the four different regimes stay qualitatively the same, while there are obviously quantitative differences now due to the greater substitutability between consumption and the environment.

In terms of the dynamics, we find that Regimes 2 and 4 stay precisely the same. In comparison to the Leontieff case, Regime 1 in the generalized CES case is only altered by a scale factor. This scale factor implies that the capital stock may grow faster or slower in the CES case than in the Leontieff case, while its steady state level is preserved. In contrast, the steady state level of environmental quality may be larger or smaller in the CES case. Finally, Regime 3 proves difficult to study, since the CES parameter induces non-linearities which do not allow explicit analytical solutions. Hence, we compare the Leontieff case of section 3.3, with the Cobb-Douglas case, where inputs in utility are just about still complements. Again, we find that the steady state capital stock stays the same. Thus, since the Leontieff case ($\theta = -\infty$) and the Cobb-Douglas case ($\theta = 0$) span the whole range of parameters where consumption and the environment are complements, we may conjecture that the steady state capital stock is the same in any regime. The steady state in environmental quality is still proportional to the capital stock, but it may now be smaller or larger than environmental quality in the Leontieff case. Finally, also in Regime 3, we again converge monotonically to this unique steady state.

## 4 Relation to the Environmental Kuznets Curve

In the previous section we introduced a fresh look at the interaction between income, the environment and culture. We discussed some of the repercussions that this study could have for a society's investment in environmental culture along its path of economic development. In this section we take a closer look at the role that optimal investments in environmental culture may play for the relationship between the environment and economic development.

In this respect, one hypothesis that has received significant attention during the past years is the Environmental Kuznets Curve (EKC) hypothesis. This hypothesis was developed in the contributions of Grossman and Krueger (1991), Shafik and Bandyopadhyay (1992) and Panayotou (1993), and subsequently further studied both theoretically and empirically (e.g. Grossman and Krueger 1995, Schmalensee et al. 1998, Millimet et al. 2003, Dinda 2004, Bertinelli and Strobl 2005, Kijima et al. 2010, Brock and Taylor 2010). The main idea behind the EKC is that the relationship between income and pollution is inversely u-shaped. This means that poorer countries increase their wealth by living off the environment, while when they get richer their economic growth may decouple from pollution.

Several prominent explanations for the existence of the EKC hypothesis tend to get forwarded, which are either somewhat more supply side oriented or focus on the demand side. For example, Grossman and Krueger (1991) forwards a more supply-side theory, where economies are more polluting when they become more industrialized, while their pollution may decrease again when their economy shifts or transits into a service economy. Related to this is the displacement and the pollution haven hypothesis (Dinda 2004), which suggests that the more polluting labor-intensive industry gets displaced into poorer countries with lower labor costs. Komen et al. (1997) proposes that technological progress occurs with economic growth and thus developed countries may be able to rely on cleaner technology for production than poorer countries.

On the demand side, Grossman and Krueger (1995) suggest that developed countries tend to have more stringent levels of regulation simply because consumers have a higher willingness to pay for environmental protection. The role of culture or institutions has been emphasized, albeit only empirically (Ng and Wang 1993, Bhattarai and Hammig 2001).

We align ourselves more with the latter literature on culture and institutions and propose that changes in environmental culture may induce an EKC relationship between wealth and environmental quality. However, as we will also show, supply-side effects, specifically technical changes, may aid this relationship significantly.

*The general mechanism*

We now discuss how the model that we introduced above can lead to the EKC through the interaction between economic development, environmental quality and environmental culture. For simplicity, we assume that Regime 3 exists, thus $\psi_x(\psi_0, 0)\eta^{\frac{1}{1-\alpha}} > \psi_0\gamma^{\frac{1}{1-\alpha}}$ holds, and that the steady state associated with Regime 3 is unique. In this case, the reader can refer to Figure 2 for a graphical treatment.

Let us presume that a country is at an early stage of economic development, such that it is rather poor but has a high level of environmental quality. This is equivalent to assuming that the given environmental quality exceeds the maximum culturally-weighted consumption level, and society is too poor for investments in environmental culture to be worthwhile. In this case we know that Regime 2 applies, and society will focus completely on consumption and thus economic growth. This naturally leads to a reduction in environmental quality while society's wealth increases. At some point environmental quality will have diminished sufficiently enough so that society sees a need to invest in abatement. As is most easily visible through Figure 2, only a sufficiently slow increase in wealth and a sufficiently fast reduction in environmental quality will lead to an EKC with a visible inverse u-shape. Once society then accumulated enough wealth to make investments in environmental culture worthwhile, this further enhances the positive feedbacks between increasing wealth and environmental quality, so that the prior reduction in environmental quality for low levels of wealth transits into a positive relationship between wealth and economic quality. Whether environmental quality simply enhances or causes this EKC-type relationship depends on whether Regime 2 transits into Regime 1 (the case of slower economic growth but faster reduction in environmental quality), or transits directly into Regime 3.

In addition to the observation that the EKC can be caused by environmental culture, this crucially hinges on the assumption that the condition for an interior solution to environmental culture is actually fulfilled. If this condition is not satisfied, then a society may find it optimal to never invest in environmental culture,[27] and consequently be stuck in a low wealth-low environmental quality trap. We now show how improvements in technology may take a country out of this trap and onto a development path where environmental culture leads to positive feedbacks between environmental quality and economic development.

[27]While this may be a theoretical possibility, empirical evidence seems to point towards positive investments in environmental culture, at least for richer societies, see the references in section 2.

*Improvements in technology*

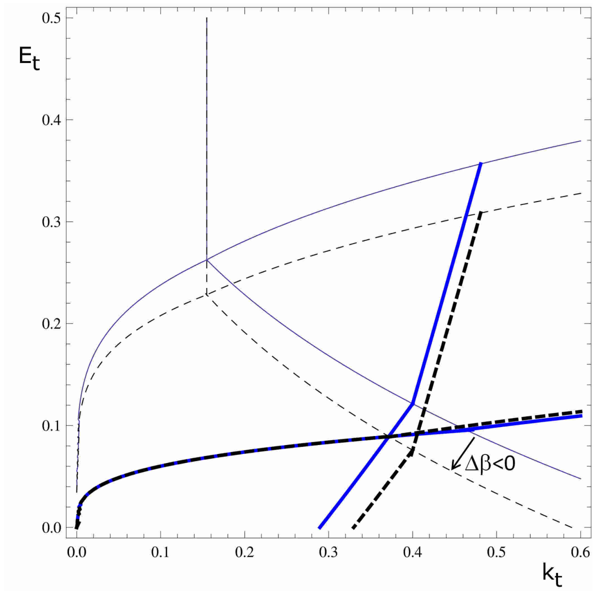
In this section we investigate changes in the impacts of production and abatement technology.[28] In order to be able to provide a graphical understanding constrained to two dimensions we present Figures 3 and 4 for the case of $\psi_X = 0$. The dashed lines are the new curves that arise from a change in the parameter in question.

If $\beta$ decreases, then this implies a more environmentally-friendly technology that leads to lower environmental damage from consumption. An improvement in the technology that leads to fewer environmental spillovers from consumption is shown in Figure 3. The steady state curves $E = z_2(k)$ and $E = w_2(k)$ shift down, $E = z_1(k)$ stays constant while the slope of $E = w_1(k)$ increases. This leads to an increase in environmental quality and capital stock at steady state since the more efficient technology makes more wealth available that may be directed towards environmental culture or environmental quality. Both Region 3 and 4 expand in size, implying that an interior solution to environmental culture is now more likely. In our example in Figure 3, society was initially stuck in a low environmental culture trap. Consequently, a society would converge to this trap if it had initially a high level of environmental quality but a low capital stock and then increased its wealth fast or reduced its environmental quality sufficiently slowly. An improvement in environmentally-friendly technology then leads to an interior solution in environmental culture, and consequently to a steady state in Region 3. This is associated with increases in both wealth and environmental quality, and thus improvements in environmentally-friendly technology can give rise to an EKC. While this may seem very close to the transition towards to service sector as in Grossman and Krueger (1991), or to the EKC resulting from technological improvements as in Komen et al. (1997), the EKC that arises in our case is not relying on continuous improvements in technology. Instead, in our case a single, even small, technological change may be sufficient to make society start invest in environmental culture, and thereby inducing the positive feedback loops between environment and wealth that we discussed in the previous section.

One remark may be in order here. If the production technology becomes more environmentally-friendly at time $t$, then this will have no impact on the decisions of the young generation at time $t$, since $\beta$ affects a generation's choices only through the level of consumption. Consequently, a change in production technology at time $t$ will only impact the decisions of the young generation at time $t + 1$. This, furthermore, implies that, in this model of intergenerational spillovers, there are no incentives for the young generation at time $t$ to make the production technology more environmentally-friendly. Only a policy maker who, at minimum, maximizes over two periods, will have an incentive to finance improvements in the consumption spillover on environmental quality. This result indicates that e.g. costly R&D expenditure may not be undertaken in case society's benefit is only in the distant future, and

[28]The analytical derivations for these comparative statics are available in the online appendix.

**Figure 3:** Changes in $\beta$

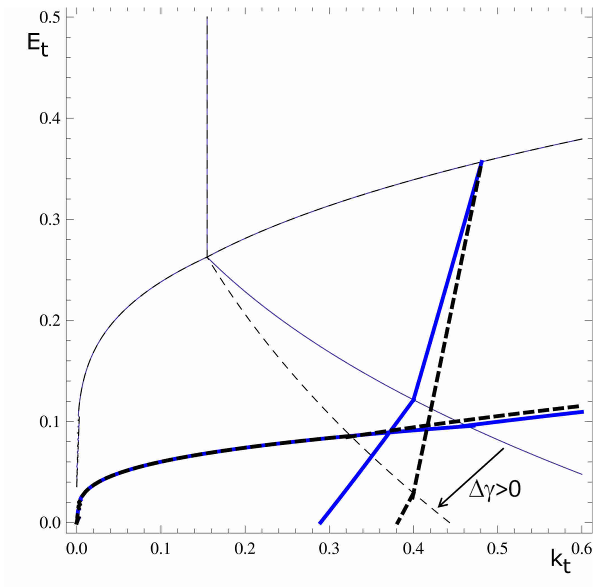



therefore society may never be able to free enough wealth to get out of the low environmental culture and environmental quality equilibrium.

An increase in $\gamma$ implies a more efficient abatement technology. This is depicted in Figure 4. The only region curve that shifts is $g_t = g_L$ which separates Region 1 and 3. Just like in the case for decreases in $\beta$, an increase in $\gamma$ shifts the steady state curves $E = z_2(k)$ and $E = w_2(k)$ down, $E = z_1(k)$ stays constant while the slope of $E = w_1(k)$ increases. In Figure 4, the improvement in abatement technology leads to an interior solution in environmental culture, and consequently to a steady state in Region 3. Again, if a society had been stuck in a low environmental culture trap, then improvements in abatement technology could make an interior solution to investments in environmental culture optimal. As a result, society may see rising levels of environmental quality with increasing environmental culture and wealth. Hence, improvements in abatement technology may give rise to an EKC, and again for different reasons than emphasized in in the technological change literature.

There are two fundamental differences between improvements in emissions per unit of consumption (i.e. reductions in $\beta$) and improvements in abatement technology (i.e. increases in $\gamma$). Firstly, changes in $\beta$ have an impact on the given level of environmental quality $g(E_t, c_t)$. This diminishes the need to improve environmental quality via maintenance, thus shrinking region 1. The increase in given environmental quality makes investments in environmental culture more profitable, thus expanding Region 3 and 4. Improvements in the effectiveness of maintenance only impact the trade-off between investments in environmental culture and abatement. Consequently, a better maintenance technology allows to direct more wealth towards environmental culture. This results in a shrinking of Region 1 and an expansion of

**Figure 4:** Changes in $\gamma$




Region 3. Secondly, while society as modeled here has no incentives to invest in improvements in $\beta$, it would potentially want to invest in $\gamma$. In contrast to changes in $\beta$, costly R&D expenditure that affects $\gamma$ may be worthwhile to undertake for a young generation since changes in the effectiveness of abatement expenditure today impact environmental quality when old and thus affect the young generation's choices already today.

# 5 Conclusion

In this article we study the role of an endogenous environmental culture in an overlapping generation model with Leontieff preferences. Both an endogenous environmental culture as well as Leontieff preferences have seen little emphasis in the environmental economics literature. However, culture has been emphasized time and again as one of the important pillars of sustainable development, while the Leontieff preferences are believed to be an empirical regularity.

Our main findings are that for low wealth levels, society is unable to free resources for environmental culture. In this case, society will only invest in environmental maintenance if environmental quality is sufficiently low. Once society has reached a certain level of economic development, then it may optimally invest a part of its wealth in developing an environmental culture. When environmental quality and wealth are both sufficiently high, then society may find it optimal to temporarily over-invest (vis-à-vis its steady state level) in environmental culture. This is optimal until environmental quality is decreased to a level from which onwards it is worthwhile for society to also invest in environmental quality. In other words, if there is no urgent need for society to improve environmental quality, then society will either invest in an environmental culture if it can afford to do so, or not invest in case it is too poor. Technological improvements in emission or abatement efficiency both raise steady state wealth and environmental quality, and make an interior solution in environmental culture more likely.

Since investments in environmental culture raise the importance of environmental quality for utility, it becomes also more worthwhile for society to invest in maintenance due to the positive feedback between culture and the environment. As a result, environmental culture has not only a positive impact on environmental quality through lower levels of consumption, but in addition it improves the environment through maintenance expenditure for wealth-environment combinations at which, without environmental culture, no maintenance would be undertaken.

This analysis suggests that wealth drives utility through higher consumption levels and the possibility to improve environmental quality through maintenance, but after a certain level of economic development there are other aspects that are raised into focus. In our case, the particular aspect we looked at is environmental culture. A society that is rich enough will invest in both culture and the environment, and thereby end up with a higher level of utility than one that is unable to invest in environmental culture. This result suggests that, after a certain level of economic development, society may wish to undergo a social change that places cultural aspects, here environmental cultural aspects, at the forefront of decision-taking.

In terms of future research, one suggestion would be to add to this analysis the last type of capital that we neglected here, namely human capital (Becker 2009). One could then study the model within an endogenous growth setting and analyze further the interaction between the different types of capital along the path of economic development. Another direction would be to take this model to the empirical literature. In this regards, it seems very reasonable to study whether environmental culture may be the main driver of an Environmental Kuznets Curve instead of wealth. In relation to this, another implication is obviously that our focus on Gross Domestic Product as a means of assessing happiness is incomplete. The results here show that while we should take consumption into account, we also need to include measures like environmental quality and culture. Only by having this holistic approach to measuring progress will we be actually able to measure our society's development.

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## Appendix

**Proof of Proposition 1** *Given the maximization problem as defined by eq.* (3) *to* (6)*, we denote an optimal, regime-dependent solution by* $x^*_{it}$*,* $a^*_{it}$ *and* $s^*_{it}$*, for* $i = 1, ..., 4$*. Substituting the constraints into* $E_{t+1} = \psi(X_t, x_t)c_{t+1}$ *gives the reduced-form utility function, which we denote by*

$$W(x_t) = \frac{\psi(X_t, x_t)}{\gamma + \psi(X_t, x_t)R_{t+1}}\Big(\gamma(w_t - x_t) + g(E_t, c_t)\Big), \tag{46}$$

*where the choice variable is* $x_t$*, which implicitly determines* $a^*_{it}$ *and* $s^*_{it}$*. Maximizing function* $W(x_t)$ *subject to* $x_t$ *gives rise to the first-order condition in eq.* (12)*. The second-order condition is given by equation* (13)*. It is negative and thus assures a maximum.*

*We define* $A(x) \equiv \psi_x(X_t, x_t)\Big(\gamma(w_t - x_t) + g(E_t, c_t)\Big)$ *and* $B(x) \equiv \psi(X_t, x_t)\big(\gamma + \psi(X_t, x_t)R_{t+1}\big)$*. These have the properties that* $A(0) = \psi_x(X_t, 0)\big(\gamma w_t + g(E_t, c_t)\big) > 0$*, while* $A(w_t) = \psi_x(X_t, w_t)g(E_t, c_t) > 0$*, with* $A_x(x_t) = \psi_{xx}(X_t, x_t)\big(\gamma(w_t - x_t) + g(E_t, c_t)\big) - \psi_x(X_t, x_t)\gamma < 0$*. In contrast,* $B(0) = \psi(X_t, 0)\big(\gamma + \psi(X_t, 0)R_{t+1}\big) > 0$*,* $B(w_t) = \psi(X_t, w_t)\big(\gamma + \psi(X_t, w_t)\big) > 0$ *with* $B(w_t) > B(0)$*, and* $B_x(x_t) = \psi_x(X_t, x_t)\big(\gamma + 2\psi(X_t, x_t)\big) > 0$*.*

*Thus, there exists a unique solution to eq.* (12) *if* $A(0) > B(0)$ *and* $B(w_t) > A(w_t)$*. Condition* $A(0) > B(0)$ *can be re-written as*

$$g(E_t, c_t) > \frac{\psi(X_t, 0)}{\psi_x(X_t, 0)}(\gamma + \psi(X_t, 0)R_{t+1}) - \gamma w_t \equiv g_L,$$

*while* $A(w_t) < B(w_t)$ *leads to*

$$g(E_t, c_t) < \frac{\psi(X_t, w_t)}{\psi_x(X_t, w_t)}(\gamma + \psi(X_t, w_t)R_{t+1}) \equiv g_H.$$

*There exists a* $g(E_t, c_t)$ *such that* $g(E_t, c_t) \in (g_L, g_H)$ *since* $g_L < g_H$ *holds always if Assumption 1 applies.*

*Solving eq.* (12) *for* $x_t$ *gives the solution* $x_t = x^*_{3t}$*. Substituting this* $x^*_{3t}$ *into eq.* (8) *then leads to the following conditions. If* $\Gamma(x^*_{3t}) > g(E_t, c_t)$ *then* $a^*_{3t} = \frac{\Gamma(x^*_{3t}) - g(E_t, c_t)}{\psi(X_t, x^*_{3t})R_{t+1} + \gamma}$*, with* $s^*_{3t} = w_t - a^*_{3t} - x^*_{3t}$ *(**Regime 3**).*

*If* $g(E_t, c_t) \in (g_L, g_H)$ *and* $\Gamma(x^*_{3t}) \leq g(E_t, c_t)$ *then* $a^*_{4t} = 0$ *and* $s^*_{4t} = w - x^*_{4t}$*. Since* $\Gamma(x^*_{3t})$ *is also implicitly a function of* $g(E_t, c_t)$*, then we need to know the position of* $\Gamma(x^*_{3t})$ *relative to* $\Gamma(x^m_t)$ *and* $g_H$ *when* $\Gamma(x^*_{3t}) \leq g(E_t, c_t)$*. This we do as follows. We take the first-order condition eq* (12) *and set* $g(E_t, c_t) = \Gamma(x^*_{3t})$*. This gives us*

$$\psi_x(X_t, x^*_{3t})\Big(\gamma(w_t - x^*_{3t}) + \psi(X_t, x^*_{3t})R_{t+1}(w_t - x^*_{3t})\Big) = \psi(X_t, x^*_{3t})\big(\gamma + \psi(X_t, x_{3t^*})R_{t+1}\big).$$

*Simplifying leads to* $\psi_x(X_t, x^*_{3t})(w_t - x^*_{3t}) = \psi(X_t, x^*_{3t})$*. This only holds if* $x^*_{3t} = x^m_t$*. There exist two sub-cases which are based on Lemma 1. An interior solution to* $\psi_x(X_t, x^*_{3t})(w_t -$

$x_{3t}^*) = \psi(X_t, x_{3t}^*)$ *requires* $w_t > \psi(X_t, 0)/\psi_x(X_t, 0)$, *and we denote the optimal solution in this case as* $x_{4t}^* = x_t^m$. *Thus, for* $\Gamma(x_{3t}^*) \leq g(E_t, c_t)$ *and* $w_t > \psi(X_t, 0)/\psi_x(X_t, 0)$ *we have that* $x_{4t}^* = x_t^m$ ***(Regime 4)****. If* $w_t \leq \psi(X_t, 0)/\psi_x(X_t, 0)$, *then there exists no interior solution to* $\psi_x(X_t, x_{3t}^*)(w_t - x_{3t}^*) = \psi(X_t, x_{3t}^*)$ *and the optimal* $x_t^*$ *will be equal to zero. We denote this as* $x_{2t}^*$ ***(Regime 2)****.*

*For* $A(0) \leq B(0)$, *then this implies* $g(E_t, c_t) \leq g_L$. *Hence,* $W_x(x_t) < 0$, $\forall x_t \leq w_t$, *and thus* $x_t^* = 0$ *and* $a_t^* \geq 0$, *where* $a_t^*$ *solves eq.* (8) *with* $x_t^* = 0$ *and* $s_t^* = w - a_t^*$. *If* $g(E_t, c_t) < \psi(X_t, 0)R_{t+1}w_t$ *then from eq.* (8) *at* $x_{1t}^* = 0$ *we obtain* $a_{1t}^* = \frac{\psi(X_t,0)R_{t+1}w_t - g(E_t,c_t)}{\gamma + \psi(X_t,0)R_{t+1}}$ ***(Regime 1****), while* $a_{2t}^* = 0$ *if* $g(E_t, c_t) \geq \psi(X_t, 0)R_{t+1}w_t$ ***(Regime 2****).* ■

**Proof of Proposition 2** *The steady state of this system can easily be calculated by solving the dynamic system* (13) *and* (15) *for their fixed points* $k$, $E$ *and* $X$. *We denote the steady state values in regime 1 by* $\bar{k}_1$, $\bar{E}_1$ *and* $\bar{X}_1$. *Solving for these gives the two steady states*

$$\bar{k}_1 = \left(\frac{\eta}{\gamma}\right)^{\frac{1}{1-\alpha}}, \tag{47}$$

$$\bar{E}_1 = \alpha\psi_0 \left(\frac{\eta}{\gamma}\right)^{\frac{\alpha}{1-\alpha}}, \tag{48}$$

$$\bar{X}_1 = \psi_0. \tag{49}$$

*For stability we linearize system* (13) *and* (15) *around the unique steady state and obtain the system*

$$\begin{bmatrix} k_{t+1} \\ E_{t+1} \\ X_{t+1} \end{bmatrix} = \begin{bmatrix} \bar{k}_1 \\ \bar{E}_1 \\ \bar{X}_1 \end{bmatrix} + \begin{bmatrix} \frac{\alpha\eta}{\psi_0\alpha^2+\eta} & \frac{\eta}{\gamma\psi_0\alpha^2+\gamma\eta} & -\frac{\alpha\left(\gamma^{-\alpha}\eta\right)^{\frac{1}{1-\alpha}}\psi_X}{\gamma\psi_0\alpha^2+\gamma\eta} \\ \frac{\alpha^3\gamma\psi_0}{\psi_0\alpha^2+\eta} & \frac{\alpha^2\psi_0}{\psi_0\alpha^2+\eta} & \frac{\alpha\left(\gamma^{-\alpha}\eta\right)^{\frac{1}{1-\alpha}}\psi_X}{\psi_0\alpha^2+\eta} \\ 0 & 0 & \psi_X \end{bmatrix} \begin{bmatrix} k_t - \bar{k}_1 \\ E_t - \bar{E}_1 \\ X_t - \bar{X}_1 \end{bmatrix}.$$

*This gives rise to the eigenvalues* $\lambda_{1a} = 0$,$\lambda_{1b} = \psi_X(\psi_0, 0) \in [0, 1)$ *by definition, and* $\lambda_{1c} = \frac{\alpha(\eta+\alpha\psi_0)}{\eta+\alpha^2\psi_0}$. *It is then straight-forward to see that* $\lambda_{1c} \in (0, 1)$, *which implies a monotonic convergence to the steady state.* ■

**Proof of Proposition 3** *Function* $\zeta(k)$ *has the characteristics* $\zeta_k(k) = \alpha\eta/\gamma k^{\alpha-1} - 1$, $\zeta(0) = 0$, *and* $\zeta\big((\frac{\eta}{\gamma})^{\frac{1}{1-\alpha}}\big) = 0$. *Then,* $\psi\big(\tilde{\psi}(\zeta(k), \zeta(k)\big)$ *can be characterized as* $\psi\big(\tilde{\psi}(\zeta(0), \zeta(0)\big) = \psi_0$, $\psi\big(\tilde{\psi}(\zeta(\frac{\eta}{\gamma})^{\frac{1}{1-\alpha}}), \zeta(\frac{\eta}{\gamma})^{\frac{1}{1-\alpha}})\big) = \psi_0$, *and it is first increasing and then decreasing. Function* $\psi_x\big(\tilde{\psi}(\zeta(k)), \zeta(k)\big)k$ *starts at zero and cuts the* $\psi\big(\tilde{\psi}(\zeta(k), \zeta(k)\big)$ *curve at least once if* $\psi_x(\psi_0, 0)\eta^{\frac{1}{1-\alpha}} > \psi_0\gamma^{\frac{1}{1-\alpha}}$ *holds.*■

**Proof of Proposition 4** *First we show that* $g_L < g_H$. *Combining and replacing the explicit functional forms gives, after simplification,* $-\alpha(2\psi_0 k^{\alpha-1} + \psi_1(1-\alpha)k^{2\alpha-1}) < 2\gamma$. *This thus holds at any interior* $k$.

*We then look at equation* (41)*. If* $\left(\psi_0+\psi_1\left[\frac{\eta}{\gamma}k^\alpha-k\right]\right)\alpha k^{\alpha-1}\neq-\gamma$*, then the equation describing the steady state is given by*

$$\psi_1 k=\psi_0+\psi_1\left[\frac{\eta}{\gamma}k^\alpha-k\right]. \tag{50}$$

*If instead equation* (50) *does not hold, then the steady state equation is defined by*

$$\alpha\left(\psi_0+\psi_1\left[\frac{\eta}{\gamma}k^\alpha-k\right]\right)=-\gamma k^{1-\alpha}. \tag{51}$$

*A third possibility is that both equations* (50) *and* (51) *hold with equality. This is the easiest to dismiss since combining both equations leads to* $\alpha\psi_1 k^\alpha=-\gamma$ *which is impossible. We now need to study whether the two potential steady states exist within the bounds given by Proposition 1. For equation* (51) *it is easy to show that* $g>g_L$ *never holds. Substituting equation* (51) *into the threshold condition* $g_t>g_L$ *gives, after some simplifications,*

$$\eta k^\alpha-\gamma k>\frac{\psi_0}{\psi_1}\left(\gamma+\psi_0\alpha k^{\alpha-1}\right).$$

*The right-hand side is positive, while the left-hand side is negative at the steady state described by equation* (51)*. Thus, the solution for k given by equation* (51) *will not define a steady state for case 3.*

*For equation* (50) *we can show that* $g<g_H$ *if* $k>\psi_0/\psi_1$*. Assuming* $g<g_H$ *requires*

$$\left(\psi_0+\psi_1\left[\frac{\eta}{\gamma}k^\alpha-k\right]\right)\alpha k^\alpha-\alpha\beta k^\alpha< \tag{52}$$

$$\frac{\psi_0+\psi_1(1-\alpha)k^\alpha}{\psi_1}\left(\gamma+(\psi_0+\psi_1(1-\alpha)k^\alpha)\alpha k^{\alpha-1}\right). \tag{53}$$

*We then collect terms and slightly rewrite to get*

$$(\psi_0+\psi_1(1-\alpha)k^\alpha)(\psi_1\alpha k^{\alpha-1}-\gamma k-(\psi_0+\psi_1(1-\alpha)k^\alpha)\alpha k^\alpha)<-\Psi,$$

*where* $\Psi\equiv-\psi_1^2(\alpha\beta/\gamma k^\alpha-k)\alpha k^\alpha-\psi_1\alpha\beta k^\alpha<0$*. Dividing through by the positive term* $(\psi_0+\psi_1(1-\alpha)k^\alpha)$ *and substituting equation* (50) *for* $k^\alpha$*, multiplying by* $\eta(>0)$ *leads to*

$$\psi_1\alpha\eta k^{\alpha+1}-\eta\gamma k-\psi_0\alpha\eta k^\alpha-\alpha(1-\alpha)\gamma(2\psi_1 k-\psi_0)k^\alpha<\tilde{\Psi}k,$$

*where* $\tilde{\Psi}=\frac{\eta}{\psi_0+\psi_1(1-\alpha)k^\alpha}\Psi$*. Collecting terms and simplifying gives*

$$-\alpha\beta(\psi_1 k-\psi_0)k^\alpha-\eta k-\alpha(1-\alpha)\gamma\psi_1 k^{\alpha+1}<\tilde{\Psi}k.$$

*A sufficient condition for this inequality to hold is* $k>\psi_0/\psi_1$*. Finally, we derive the condition* *under which equation* (50) *implies* $g > g_L$. *Assuming* $g > g_L$ *and substituting equation* (50) *gives*

$$\psi_1 \alpha k^{\alpha+1} > \frac{\psi_0}{\psi_1}(\gamma + \psi_0 \alpha k^{\alpha-1}) - \eta k^{\alpha}.$$

*Slightly rewriting gives us*

$$\eta + \psi_1 \alpha k > \frac{\psi_0}{\psi_1}(\gamma k^{-\alpha} + \psi_0 \alpha k^{-1}).$$

*Substituting equation* (50) *again gives*

$$\eta \psi_1 k \left(2 \frac{\psi_1 k - \psi_0}{2\psi_1 k - \psi_0}\right) + \psi_1^2 \alpha k^2 > \psi_0^2 \alpha.$$

*Clearly, if* $k > \psi_0/\psi_1$, *then this inequality is satisfied. If* $k \in \left[\frac{\psi_0}{2\psi_1}, \frac{\psi_0}{\psi_1}\right)$, *then* $g < g_L$. *Finally, define* $\Omega(k) \equiv \eta k \left(2 \frac{\psi_1 k - \psi_0}{2\psi_1 k - \psi_0}\right) + \psi_1^2 \alpha k^2$. *Then it is easy to show that* $\Omega'(k) > 0$ *if* $k < \frac{\psi_0}{2\psi_1}$. *Thus, we conclude that* $g < g_L$, $\forall k < \psi_0/\psi_1$.

*For stability, we proceed as follows. We linearize system* (37) *and* (38) *around the unique steady state to obtain the system*

$$\begin{bmatrix} E_{t+1} \\ k_{t+1} \end{bmatrix} = \begin{bmatrix} E \\ k \end{bmatrix} + DH(E,k) \begin{bmatrix} E_t - E \\ k_t - k \end{bmatrix},$$

*where matrix* $DH(E,k)$ *takes the form*

$$DH(E,k) = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}.$$

*The individual elements of matrix* $DH(E,k)$ *are given by*

$$H_{11} = \frac{k^{\alpha} \alpha (1+\alpha) \psi_1}{2\gamma + k^{\alpha} \alpha (1+\alpha) \psi_1},$$

$$H_{12} = \frac{k^{-1+2\alpha} \alpha^2 (1+\alpha) \gamma \psi_1 (\eta - \alpha(\psi_0 - 2k\psi_1))}{(\gamma + k^{\alpha} \alpha \psi_1)(2\gamma + k^{\alpha} \alpha (1+\alpha) \psi_1)},$$

$$H_{21} = \frac{1}{2\gamma + k^{\alpha} \alpha (1+\alpha) \psi_1},$$

$$H_{22} = \frac{\alpha \eta}{k (2k^{-\alpha} \gamma + \alpha (1+\alpha) \psi_1)}.$$

*From this we obtain the characteristic polynomial*

$$\lambda^2 - \frac{\alpha(\eta + k\psi_1 + \alpha k \psi_1)}{k (2k^{-\alpha} \gamma + \alpha (1+\alpha) \psi_1)} \lambda = 0 \qquad (54)$$

*This gives rise to the eigenvalues* $\lambda_1 = 0$ *and* $\lambda_2 = \frac{\alpha(\eta + k\psi_1 + \alpha k \psi_1)}{k\left(\alpha(1+\alpha)\psi_1 + \frac{2\eta\psi_1}{2k\psi_1 - \psi_0}\right)}$. *Then* $\lambda_2 < 1$ *implies* $\psi_0 - 2k\psi_1 < 2\eta$, *which holds under Assumption 4 and iff* $k > \psi_0/\psi_1$; *and* $\lambda_2 > -1$ *implies* $\alpha\eta > -2k\psi_1\left(\alpha(1-\alpha) + \frac{\eta}{2k\psi_1 - \psi_0}\right)$. *Again, this holds if* $k > \psi_0/\psi_1$ *and under Assumption 4. Finally, it is easy to see that* $\lambda_2 > 0$. *Thus, there are no cycles and convergence to the steady state is monotonic.* ■

**Proof of Proposition 5** *The first part of proposition follows directly from proof 1. The second part can be proven as follows. Define*

$$B(k) = \psi_1\eta/\gamma k^{\alpha} + \psi_0 - 2\psi_1 k.$$

*Since function* $B(k)$ *is positive for* $k < \bar{k}_3$ *and negative for* $k > \bar{k}_3$, *then it follows that* $\hat{k} > \bar{k}_3$ *if* $B(\bar{k}) < 0$. *Substituting the solution for* $\bar{k} = (\eta/\gamma)^{\frac{1}{1-\alpha}}$ *into* $B(k)$ *and assuming that* $B(k) < 0$ *yields*

$$B(\hat{k}) = \psi_1\eta/\gamma(\eta/\gamma)^{\frac{\alpha}{1-\alpha}} + \psi_0 - 2\psi_1(\eta/\gamma)^{\frac{1}{1-\alpha}} < 0.$$

*Simplifying gives the condition* $\psi_0/\psi_1 < (\eta/\gamma)^{\frac{1}{1-\alpha}} \equiv \hat{k}$. *At this parameter configuration we know that Regime 3 applies, and consequently there is no contradiction. Thus* $\hat{k} > \bar{k}_3$.

*Assume now that* $\hat{E} < \bar{E}_3$, *thus*

$$\bar{E}_3 = \alpha\psi_1\bar{k}_3^{1+\alpha} > \alpha\psi_0\bar{k}^{\alpha}.$$

*This implies* $\bar{k}_3 > (\psi_0/\psi_1)^{\frac{1}{1+\alpha}}(\eta/\gamma)^{\frac{\alpha}{1-\alpha^2}} \equiv \tilde{k}$. *If* $B(\tilde{k}) > 0$, *then* $\bar{k}_3 > \tilde{k}$ *and consequently* $\hat{E} < \bar{E}_3$. *Evaluating* $B(\tilde{k}) > 0$ *gives the condition*

$$(\psi_0/\psi_1)^{\frac{\alpha}{1+\alpha}}(\eta/\gamma)^{\frac{1}{1-\alpha^2}} + \psi_0/\psi_1 - 2(\psi_0/\psi_1)^{\frac{1}{1+\alpha}}(\eta/\gamma)^{\frac{\alpha}{1-\alpha^2}} > 0.$$

*This is equivalent to*

$$\hat{k}^{\frac{1}{1+\alpha}} + (\psi_0/\psi_1)^{\frac{1}{1+\alpha}} - 2(\psi_0/\psi_1)^{\frac{1-\alpha}{1+\alpha}}\hat{k}^{\frac{\alpha}{1+\alpha}} > 0.$$

*This condition holds if* $1 - 2(\psi_0/\psi_1)^{\frac{1-\alpha}{1+\alpha}}\hat{k}^{\frac{\alpha-1}{1+\alpha}}$, *which can be re-written to* $\hat{k} > 2^{\frac{1+\alpha}{1-\alpha}}\psi_0/\psi_1$. *This holds always in Regime 3 since* $2^{\frac{1+\alpha}{1-\alpha}} > 1$ *and* $\frac{\hat{k} > \psi_0}{\psi_1}$. *Thus, we find that* $\hat{E} < \bar{E}_3$. ■

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